8-1 Study Guide and Intervention

Multiplying and Dividing Rational Expressions

Simplify Rational Expressions A ratio of two polynomial expressions is a rational expression. To simplify a rational expression, divide both the numerator and the denominator by their greatest common factor (GCF).

<table>
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<tr>
<th>Multiplying Rational Expressions</th>
<th>For all rational expressions ( \frac{a}{b} ) and ( \frac{c}{d} ), ( \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} ), if ( b \neq 0 ) and ( d \neq 0 ).</th>
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<td>Dividing Rational Expressions</td>
<td>For all rational expressions ( \frac{a}{b} ) and ( \frac{c}{d} ), ( \frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc} ) if ( b \neq 0 ), ( c \neq 0 ), and ( d \neq 0 ).</td>
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Example

Simplify each expression.

a. \( \frac{24a^3b^2}{(2ab)^4} \)

\[
\frac{24a^3b^2}{(2ab)^4} = \frac{1}{2^4} \cdot \frac{2^4 \cdot a^4 \cdot b^4}{2^4 \cdot a^4 \cdot b^4} = \frac{3a}{2b^2}
\]

b. \( \frac{3r^m}{5t^3} \cdot \frac{20t^2}{9r^n} \)

\[
\frac{3r^m}{5t^3} \cdot \frac{20t^2}{9r^n} = \frac{1}{5} \cdot \frac{20 \cdot r^m \cdot t^2 \cdot r^m}{18 \cdot r^n \cdot t^3} = \frac{2r}{3 \cdot r \cdot t^t} = \frac{4n^2}{3r^t}
\]

c. \( \frac{x^2 + 8x + 16}{2x - 2} \div \frac{x^2 + 2x - 8}{x - 1} \)

\[
\frac{x^2 + 8x + 16}{2x - 2} \div \frac{x^2 + 2x - 8}{x - 1} = \frac{x^2 + 8x + 16}{2x - 2} \cdot \frac{x - 1}{x^2 + 2x - 8}
\]

\[
= \frac{1}{2} \cdot \frac{(x + 4)(x + 4)(x - 1)}{2(x - 1)(x - 2)(x + 4)} = \frac{x + 4}{2(x - 2)}
\]

Exercises

Simplify each expression.

1. \( \frac{(-2ab)^3}{20ab^4} \)

2. \( \frac{4x^2 - 12x + 9}{9 - 6x} \)

3. \( \frac{x^2 + x - 6}{x^2 - 6x - 27} \)

4. \( \frac{3m^3 - 3m}{6m^4} \cdot \frac{4m^5}{m + 1} \)

5. \( \frac{c^2 - 3c}{c^2 - 25} \cdot \frac{c^2 + 4c - 5}{c^2 - 4c + 3} \)

6. \( \frac{(m - 3)^2}{m^2 - 6m + 9} \div \frac{m^3 - 9m}{m^2 - 9} \)

7. \( \frac{6xy^4}{25x^3} \div \frac{18x^2}{5y} \)

8. \( \frac{16p^2 - 8p + 1}{14p^4} \div \frac{4p^2 + 7p - 2}{7p^5} \)

9. \( \frac{2m - 1}{m^2 - 3m - 10} \div \frac{4m^2 - 1}{4m + 8} \)
Simplify Complex Fractions  A complex fraction is a rational expression with a numerator and/or denominator that is also a rational expression. To simplify a complex fraction, first rewrite it as a division problem.

Example

Simplify \( \frac{\frac{3n - 1}{n}}{\frac{3n^2 + 8n - 3}{n^4}} \).

\[
\frac{\frac{3n - 1}{n}}{\frac{3n^2 + 8n - 3}{n^4}} = \left( \frac{3n - 1}{n} \right) \div \left( \frac{3n^2 + 8n - 3}{n^4} \right) \\
= \frac{3n - 1}{n} \cdot \frac{n^4}{3n^2 + 8n - 3} \\
= \frac{(3n - 1)n^3}{n(3n^2 - 1)(n + 3)} \\
= \frac{n^3}{n + 3}
\]

Exercises

Simplify each expression.

1. \( \frac{x^3y^2z}{a^2b^2} \div \frac{a^3x^2y}{b^2} \)
2. \( \frac{a^2bc^3}{x^2y} \div \frac{ab^2}{c^x} \)
3. \( \frac{b^2 - 1}{3b + 1} \div \frac{b + 1}{3b^2 - b - 2} \)

4. \( \frac{b^2 - 100}{b^2} \div \frac{3b^2 - 31b + 10}{2b} \)
5. \( \frac{x - 4}{x^2 + 6x + 9} \div \frac{x^2 + 2x - 8}{3 + x} \)

6. \( \frac{a^2 - 16}{a + 2} \div \frac{a^2 + 3a - 4}{a^2 + a - 2} \)
7. \( \frac{2x^2 + 9x + 9}{x + 1} \div \frac{10x^2 + 19x + 6}{5x^2 + 7x + 2} \)

8. \( \frac{b + 2}{b^2 - 6b + 8} \div \frac{b^2 - b - 2}{b^2 - 16} \)
9. \( \frac{x^2 - x - 2}{x^2 + x - 6} \div \frac{x + 1}{x + 3} \)
8-1 Skills Practice

Multiplying and Dividing Rational Expressions

Simplify each expression.

1. \( \frac{21x^3y}{14x^2y^2} \)

2. \( \frac{5ab^3}{25a^2b^2} \)

3. \( \frac{x^5y^3}{(x^2)^4} \)

4. \( \frac{8y^2(y^6)^3}{4y^{24}} \)

5. \( \frac{18}{2x - 6} \)

6. \( \frac{x^2 - 4}{(x - 2)(x + 1)} \)

7. \( \frac{3a^2 - 24a}{3a^2 + 12a} \)

8. \( \frac{3m}{2f} \cdot \frac{f^3}{6} \)

9. \( \frac{24g^3}{5f^2} \cdot \frac{10(gf)^3}{8g^2f} \)

10. \( \frac{5r^2}{r^2 - 4} \cdot \frac{r + 2}{10r^5} \)

11. \( \frac{7g}{y^2} \div 21g^3 \)

12. \( \frac{80y^4}{49x^3y^7} \div \frac{25y^5}{14x^2z^6} \)

13. \( \frac{3x^2}{x + 2} \div \frac{3x}{x^2 - 4} \)

14. \( \frac{q^2 + 2q}{6q} \div \frac{q^2 - 4}{3q^2} \)

15. \( \frac{w^2 - 5w - 24}{w + 1} \div \frac{w^2 - 6w - 7}{w + 3} \)

16. \( \frac{t^2 + 19t + 84}{4t - 4} \div \frac{2t - 2}{t^2 + 9t + 14} \)

17. \( \frac{x^2 - 5x + 4}{2x - 8} \div (3x^2 - 3x) \)

18. \( \frac{16a^2 + 40a + 25}{3a^2 - 10a - 8} \div \frac{4a + 5}{a^2 - 8a + 16} \)

19. \( \frac{c^2y}{2a^2} \div \frac{-c^6}{5d} \)

20. \( \frac{a^2 - b^2}{a + b} \div \frac{4a}{2a} \)
8-1 Practice

Multiplying and Dividing Rational Expressions

Simplify each expression.

1. \(\frac{9a^2b^3}{27a^5b^4c}\)

2. \(\frac{(2m^n)^3}{-18m^n}\)

3. \(\frac{10y^2 + 15y}{35y^2 - 5y}\)

4. \(\frac{2k^2 - k - 15}{k^2 - 9}\)

5. \(\frac{25 - v^2}{3v^2 - 13v - 10}\)

6. \(\frac{x^4 + x^3 - 2x^2}{x^4 - x^3}\)

7. \(\frac{-2u^3y}{15x^2} \cdot \frac{25x^3}{14u^2y^2}\)

8. \(\frac{a + y}{6} \cdot \frac{4}{y + a}\)

9. \(\frac{n^5}{n - 6} \cdot \frac{n^2 - 6n}{n^8}\)

10. \(\frac{a - y}{w + n} \cdot \frac{w^2 - n^2}{y - a}\)

11. \(\frac{x^2 - 5x - 24}{6x + 2x^2} \cdot \frac{5x^2}{8 - x}\)

12. \(\frac{x - 5}{10x - 2} \cdot \frac{25x^2 - 1}{x^2 - 10x + 25}\)

13. \(\frac{a^5y^3}{wy^7} \div \frac{a^7w^2}{w^3y^2}\)

14. \(\frac{2xy}{w^2} \div \frac{24x^2}{w^5}\)

15. \(\frac{x + y}{6} \div \frac{x^2 - y^2}{3}\)

16. \(\frac{3x + 6}{x^2 - 9} \div \frac{6x^2 + 12x}{4x + 12}\)

17. \(\frac{2s^3 - 7s - 15}{(s + 4)^2} \div \frac{s^2 - 10s + 25}{s + 4}\)

18. \(\frac{9 - a^2}{a^2 + 5a + 6} \div \frac{2a - 6}{5a + 10}\)

19. \(\frac{2x + 1}{4 - x} \div \frac{x}{x}\)

20. \(\frac{x^2 - 9}{3 - x} \div \frac{4}{8}\)

21. \(\frac{x^3 + 2^3}{x^2 - 2x} \div \frac{(x + 2)^3}{x^2 + 4x + 4}\)

22. GEOMETRY A right triangle with an area of \(x^2 - 4\) square units has a leg that measures \(2x + 4\) units. Determine the length of the other leg of the triangle.

23. GEOMETRY A rectangular pyramid has a base area of \(\frac{x^2 + 3x - 10}{2x}\) square centimeters and a height of \(\frac{x^2 - 3x}{x^2 - 5x + 6}\) centimeters. Write a rational expression to describe the volume of the rectangular pyramid.
8-1 Word Problem Practice

Multiplying and Dividing Rational Expressions

1. JELLY BEANS A large vat contains $G$ green jelly beans and $R$ red jelly beans. A bag of 100 red and 100 green jelly beans is added to the vat. What is the new ratio of red to green jelly beans in the vat?

2. MILEAGE Beth drives a hybrid car that gets 45 miles per gallon in the city and 48 miles per gallon on the highway. Beth uses $C$ gallons of gas in the city and $H$ gallons of gas on the highway. Write an expression for the average number of miles per gallon that Beth gets with her car in terms of $C$ and $H$.

3. HEIGHT The front face of a Nordic house is triangular. The surface area of the face is $x^2 + 3x + 10$ where $x$ is the base of the triangle.

4. OIL SLICKS David was moving a drum of oil around his circular outdoor pool when the drum cracked, and oil spilled into the pool. The oil spread itself evenly over the surface of the pool. Let $V$ denote the volume of oil spilled and let $r$ be the radius of the pool. Write an equation for the thickness of the oil layer.

5. RUNNING Harold runs to the local food mart to buy a gallon of soy milk. Because he is weighed down on his return trip, he runs slower on the way back. He travels $S_1$ feet per second on the way to the food mart and $S_2$ feet per second on the way back. Let $d$ be the distance he has to run to get to the food mart. Remember: distance = rate $\times$ time.

  a. Write an equation that gives the total time Harold spent running for this errand.

  b. What speed would Harold have to run if he wanted to maintain a constant speed for the entire trip yet take the same amount of time running?
8-1 Enrichment

Dimensional Analysis

Scientists always express the units of measurement in their solution. It is insufficient and ambiguous to state a solution regarding distance as 17; Seventeen what, feet, miles, meters? Often it is helpful to analyze the units of the quantities in a formula to determine the desired units of an output. For example, it is known that torque is the product of force and distance, but what are the units of force?

The units also depend on the measuring system. The two most commonly used systems are the British system and the international system of units (SI). Some common units of the British system are inches, feet, miles, and pounds. Common SI units include meters, kilometers, Newtons, and grams. Frequently conversion from one system to another is necessary and accomplished by multiplication by conversion factors.

Consider changing units from miles per hour to kilometers per hour. What is 60 miles per hour in kilometers per hour? Use the conversion 1 ft = 30.5 cm.

\[
60 \text{ mi/h} = 60 \text{ mi/h} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{30.5 \text{ cm}}{1 \text{ ft}} \times \frac{1 \text{ m}}{100 \text{ cm}} \times \frac{1 \text{ km}}{1000 \text{ m}} = 96.62 \text{ km/h}
\]

1. The SI unit for force is a Newton (N) and the SI unit for distance is meters or centimeters. The British unit for force is pounds and the British unit for distance is feet or inches. Using the formula for torque (torque = force times distance), determine the SI unit and the British unit for torque.

2. The density of a fluid is given by the formula \( \text{density} = \frac{\text{mass}}{\text{volume}} \). Suppose that a volume of a fluid in a cylindrical can is \( \pi r^2 h \), where \( r \) and \( h \) are measured in meters. Find an expression for the mass, given in kilograms (kg), of gasoline, which has a known density of \( 680 \frac{\text{kg}}{\text{m}^3} \).

3. Convert the following measurements.
   a. 72 miles/hour to feet/second
   b. 32 pounds/square inch to pounds per square foot
   c. 100 kilometers/hour to miles per hour