6-6 Study Guide and Intervention

Rational Exponents

Rational Exponents and Radicals

Definition of $b^{\frac{1}{n}}$	For any real number b and any positive integer n, $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and n is even.
Definition of $b^{\frac{m}{n}}$	For any nonzero real number b, and any integers m and n, with $n > 1$, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$, except when $b < 0$ and n is even.

Example 1 **Write $28^{\frac{1}{2}}$ in radical form.**

Notice that $28 > 0$.

$$\begin{aligned} 28^{\frac{1}{2}} &= \sqrt{28} \\ &= \sqrt{2^2 \cdot 7} \\ &= \sqrt{2^2} \cdot \sqrt{7} \\ &= 2\sqrt{7} \end{aligned}$$

Example 2 **Evaluate $\left(\frac{-8}{-125}\right)^{\frac{1}{3}}$.**

Notice that $-8 < 0$, $-125 < 0$, and 3 is odd.

$$\begin{aligned} \left(\frac{-8}{-125}\right)^{\frac{1}{3}} &= \frac{\sqrt[3]{-8}}{\sqrt[3]{-125}} \\ &= \frac{-2}{-5} \\ &= \frac{2}{5} \end{aligned}$$

Exercises

Write each expression in radical form, or write each radical in exponential form.

1. $11^{\frac{1}{7}}$

2. $15^{\frac{1}{3}}$

3. $300^{\frac{3}{2}}$

4. $\sqrt{47}$

5. $\sqrt[3]{3a^5b^2}$

6. $\sqrt[4]{162p^5}$

Evaluate each expression.

7. $-27^{\frac{2}{3}}$

8. $216^{\frac{1}{3}}$

9. $(0.0004)^{\frac{1}{2}}$

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

6-6 Study Guide and Intervention *(continued)*

Rational Exponents

Simplify Expressions All the properties of powers from Lesson 6-1 apply to rational exponents. When you simplify expressions with rational exponents, leave the exponent in rational form, and write the expression with all positive exponents. Any exponents in the denominator must be positive integers.

When you simplify radical expressions, you may use rational exponents to simplify, but your answer should be in radical form. Use the smallest index possible.

Example 1 **Simplify $y^{\frac{2}{3}} \cdot y^{\frac{3}{8}}$.**

$y^{\frac{2}{3}} \cdot y^{\frac{3}{8}} = y^{\frac{2}{3}+\frac{3}{8}} = y^{\frac{25}{24}}$

Example 2 **Simplify $\sqrt[4]{144x^6}$.**

$$\begin{aligned}\sqrt[4]{144x^6} &= (144x^6)^{\frac{1}{4}}\\ &= (2^4 \cdot 3^2 \cdot x^6)^{\frac{1}{4}}\\ &= (2^4)^{\frac{1}{4}} \cdot (3^2)^{\frac{1}{4}} \cdot (x^6)^{\frac{1}{4}}\\ &= 2 \cdot 3^{\frac{1}{2}} \cdot x^{\frac{3}{2}} = 2x \cdot (3x)^{\frac{1}{2}} = 2x\sqrt{3x}\end{aligned}$$

Exercises

Simplify each expression.

1. $x^{\frac{4}{5}} \cdot x^{\frac{6}{5}}$

2. $\left(y^{\frac{2}{3}}\right)^{\frac{3}{4}}$

3. $p^{\frac{4}{5}} \cdot p^{\frac{7}{10}}$

4. $\left(m^{\frac{6}{5}}\right)^{\frac{2}{5}}$

5. $x^{\frac{3}{8}} \cdot x^{\frac{4}{3}}$

6. $\left(s^{\frac{1}{6}}\right)^{\frac{4}{3}}$

7. $\dfrac{p}{p^{\frac{1}{3}}}$

8. $\dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}}$

9. $\sqrt[6]{128}$

10. $\sqrt[4]{49}$

11. $\sqrt[5]{288}$

12. $\sqrt{32} \cdot 3\sqrt{16}$

13. $\sqrt[3]{25} \cdot \sqrt{125}$

14. $\sqrt[6]{16}$

15. $\dfrac{a\sqrt[3]{b^4}}{\sqrt{ab^3}}$

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

6-6 Skills Practice

Rational Exponents

Write each expression in radical form, or write each radical in exponential form.

1. $3^{\frac{1}{6}}$

2. $8^{\frac{1}{5}}$

3. $\sqrt{51}$

4. $\sqrt[4]{15^3}$

5. $12^{\frac{2}{3}}$

6. $\sqrt[3]{37}$

7. $(c^3)^{\frac{3}{5}}$

8. $\sqrt[3]{6xy^2}$

Evaluate each expression.

9. $32^{\frac{1}{5}}$

10. $81^{\frac{1}{4}}$

11. $27^{\frac{1}{3}}$

12. $4^{\frac{1}{2}}$

13. $16^{\frac{3}{2}}$

14. $(-243)^{\frac{4}{5}}$

15. $27^{\frac{1}{3}} \cdot 27^{\frac{5}{3}}$

16. $\left(\frac{4}{9}\right)^{\frac{3}{2}}$

Simplify each expression.

17. $c^{\frac{12}{5}} \cdot c^{\frac{3}{5}}$

18. $m^{\frac{2}{9}} \cdot m^{\frac{16}{9}}$

19. $\left(q^{\frac{1}{2}}\right)^3$

20. $p^{\frac{1}{5}} \cdot p^{\frac{1}{2}}$

21. $x^{\frac{6}{11}} \cdot x^{\frac{4}{11}}$

22. $\dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{4}}}$

23. $\dfrac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$

24. $\dfrac{n^{\frac{1}{3}}}{n^{\frac{1}{6}} \cdot n^{\frac{1}{2}}}$

25. $\sqrt[12]{64}$

26. $\sqrt[8]{49a^8b^2}$

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

6-6 Practice

Rational Exponents

Write each expression in radical form, or write each radical in exponential form.

1. $5^{\frac{1}{3}}$ **2.** $6^{\frac{2}{5}}$ **3.** $m^{\frac{4}{7}}$ **4.** $(n^3)^{\frac{2}{5}}$

5. $\sqrt{79}$ **6.** $\sqrt[4]{153}$ **7.** $\sqrt[3]{27m^6n^4}$ **8.** $\sqrt[5]{2a^{10}b}$

Evaluate each expression.

9. $81^{\frac{1}{4}}$ **10.** $1024^{\frac{1}{5}}$ **11.** $8^{\frac{5}{3}}$

12. $-256^{\frac{3}{4}}$ **13.** $(-64)^{\frac{2}{3}}$ **14.** $27^{\frac{1}{3}} \cdot 27^{\frac{4}{3}}$

15. $\left(\frac{125}{216}\right)^{\frac{2}{3}}$ **16.** $\frac{64^{\frac{2}{3}}}{343^{\frac{2}{3}}}$ **17.** $\left(25^{\frac{1}{2}}\right)\left(-64^{-\frac{1}{3}}\right)$

Simplify each expression.

18. $g^{\frac{4}{7}} \cdot g^{\frac{3}{7}}$ **19.** $s^{\frac{3}{4}} \cdot s^{\frac{13}{4}}$ **20.** $\left(u^{\frac{1}{3}}\right)^{\frac{4}{5}}$ **21.** $y^{-\frac{1}{2}}$

22. $b^{\frac{3}{5}}$ **23.** $\frac{q^{\frac{3}{5}}}{q^{\frac{2}{5}}}$ **24.** $\frac{t^{\frac{2}{3}}}{5t^{\frac{1}{2}} \cdot t^{-\frac{3}{4}}}$ **25.** $\frac{2z^{\frac{1}{2}}}{z^{\frac{1}{2}} - 1}$

26. $\sqrt[10]{8^5}$ **27.** $\sqrt{12} \cdot \sqrt[5]{12^3}$ **28.** $\sqrt[4]{6} \cdot 3\sqrt[4]{6}$ **29.** $\frac{a}{\sqrt{3b}}$

30. ELECTRICITY The amount of current in amps I that an appliance uses can be calculated using the formula $I = \left(\frac{P}{R}\right)^{\frac{1}{2}}$, where P is the power in watts and R is the resistance in ohms. How much current does an appliance use if $P = 500$ watts and $R = 10$ ohms? Round your answer to the nearest tenth.

31. BUSINESS A company that produces DVDs uses the formula $C = 88n^{\frac{1}{3}} + 330$ to calculate the cost C in dollars of producing n DVDs per day. What is the company's cost to produce 150 DVDs per day? Round your answer to the nearest dollar.

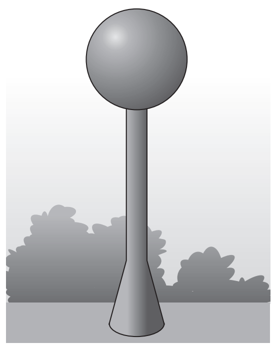
Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

6-6 Word Problem Practice

Rational Exponents

1. SQUARING THE CUBE A cube has side length s. What side length of the square will cause its area to have the same numerical value as the volume of the cube? Write your answer using rational exponents.

2. WATER TOWER Typically, drinking water for towns is stored in water towers. A water tower in Edmond, Oklahoma is 218 feet high and holds half a million gallons. One town is replacing its water tower. Residents of the town insist that their new tower be a sphere. If the new tank will hold 10 times as much water as the old tank, how many times longer should the radius of the new tank be compared to the old tank? Write your answer using rational exponents.



3. BALLOONS A spherical balloon is being inflated faster and faster. The volume of the balloon as a function of time is $9\pi t^2$. What is the radius of the balloon as a function of time? Write your answer using rational exponents.

4. INTEREST Rita opened a bank account that accumulated interest at the rate of 1% compounded annually. Her money accumulated interest in that account for 8 years. She then took all of her money out of that account and placed it into another account that paid 5% interest compounded annually. After 4 years, she took all of her money out of that account. What single interest rate when compounded annually would give her the same outcome for those 12 years? Round your answer to the nearest hundredth of a percent.

5. CELLS The number of cells in a cell culture grows exponentially. The number of cells in the culture as a function of time is given by the expression $N\left(\frac{6}{5}\right)^t$, where t is measured in hours and N is the initial size of the culture.

a. After 3 hours, there were 1728 cells in the culture. What is N?

b. How many cells were in the culture after 20 minutes? Express your answer in simplest form.

c. How many cells were in the culture after 2.5 hours? Express your answer in simplest form.

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

6-6 Enrichment

Lesser-Known Geometric Formulas

Many geometric formulas involve radical expressions.

Make a drawing to illustrate each of the formulas given on this page. Then evaluate the formula for the given value of the variable. Round answers to the nearest hundredth.

1. The area of an isosceles triangle. Two sides have length a; the other side has length c. Find A when $a = 6$ and $c = 7$.

$A = \frac{c}{4}\sqrt{4a^2 - c^2}$

2. The area of an equilateral triangle with a side of length a. Find A when $a = 8$.

$A = \frac{a^2}{4}\sqrt{3}$

3. The area of a regular pentagon with a side of length a. Find A when $a = 4$.

$A = \frac{a^2}{4}\sqrt{25 + 10\sqrt{5}}$

4. The area of a regular hexagon with a side of length a. Find A when $a = 9$.

$A = \frac{3a^2}{2}\sqrt{3}$

5. The volume of a regular tetrahedron with an edge of length a. Find V when $a = 2$.

$V = \frac{a^3}{12}\sqrt{2}$

6. The area of the curved surface of a right cone with an altitude of h and radius of base r. Find S when $r = 3$ and $h = 6$.

$S = \pi r\sqrt{r^2 + h^2}$

7. Heron's Formula for the area of a triangle uses the semi-perimeter s, where $s = \frac{a + b + c}{2}$. The sides of the triangle have lengths a, b, and c. Find A when $a = 3$, $b = 4$, and $c = 5$.

$A = \sqrt{s(s - a)(s - b)(s - c)}$

8. The radius of a circle inscribed in a given triangle also uses the semi-perimeter. Find r when $a = 6$, $b = 7$, and $c = 9$.

$r = \frac{\sqrt{s(s - a)(s - b)(s - c)}}{s}$

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.

NAME ______________________ DATE ____________ PERIOD ____________

6-6 Spreadsheet Activity

Appreciation and Depreciation

When an asset such as a house increases in value over time, it is said to *appreciate*. If the value increases by a fixed percent each year, or other period of time, the amount y of that quantity after t years is given by

$$y = a(1 + r)^t,$$

where a is the initial amount and r is the percent of increase expressed as a decimal. You can use a spreadsheet to investigate future values of an asset.

Example **Michael Blackstock is considering buying a piece of investment property for $95,000. The homes in the area are appreciating at an average rate of 4% per year. Find the expected value of the home in 1 year, 1 year and 6 months, 4 years, and 6 years and 9 months.**

Use rows 1 and 2 to enter the initial amount and the rate of increase. Then use Column A to enter the amounts of time. Enter the numbers of months as a fraction of a year since t is measured in years. Column B contains the formulas for the value of the home.

Format the cells containing the values as currency so that they are displayed as dollars and cents. The expected value of the home after each amount of time is shown in the spreadsheet.

Appreciation.xls

	A	B
1	Initial value =	$95,000.00
2	Rate =	0.04
3		
4	Years	Value
5	1	$98,800.00
6	1.5	$100,756.63
7	4	$111,136.56
8	6.75	$123,793.73

Sheet 1 Sheet 2

Exercises

1. If Mr. Blackstock chooses another property in the neighborhood that costs $99,900, what are the expected values of that home in the same periods of time?

2. What would Mr. Blackstock's profit be on the $99,900 home if he sold it after 9 years and 3 months?

3. If an antique chair worth $165.00 increases in value an average of $3\frac{1}{2}\%$ every year, how much will it be worth next year?

4. Often assets like cars decrease in value over time. This asset is said to *depreciate*. If the value decreases by a fixed percent each year, or other period of time, the amount y of that quantity after t years is given by $y = a(1 - r)^t$, where a is the initial amount and r is the percent of decrease expressed as a decimal. Use a spreadsheet to find the value of a car purchased for $18,500 after 2 years, 2 years and 6 months, and 4 years and 3 months if the car depreciates at a rate of 12% per year.

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.