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6-4 Study Guide and Intervention

*n*th Roots

Simplify Radicals

Square Root	For any real numbers a and b, if $a^2 = b$, then a is a square root of b.
***n*th Root**	For any real numbers a and b, and any positive integer n, if $a^n = b$, then a is an nth root of b.
Real *n*th Roots of *b*, $\sqrt[n]{b}$, $-\sqrt[n]{b}$	1. If n is even and $b > 0$, then b has one positive real root and one real negative root. 2. If n is odd and $b > 0$, then b has one positive real root. 3. If n is even and $b < 0$, then b has no real roots. 4. If n is odd and $b < 0$, then b has one negative real root.

Example 1 **Simplify $\sqrt{49z^8}$.**

$\sqrt{49z^8} = \sqrt{(7z^4)^2} = 7z^4$

z^4 must be positive, so there is no need to take the absolute value.

Example 2 **Simplify $-\sqrt[3]{(2a-1)^6}$**

$-\sqrt[3]{(2a-1)^6} = -\sqrt[3]{[(2a-1)^2]^3} = -(2a-1)^2$

Exercises

Simplify.

1. $\sqrt{81}$

2. $\sqrt[3]{-343}$

3. $\sqrt{144p^6}$

4. $\pm\sqrt{4a^{10}}$

5. $\sqrt[5]{243p^{10}}$

6. $-\sqrt[3]{m^6n^9}$

7. $\sqrt[3]{-b^{12}}$

8. $\sqrt{16a^{10}b^8}$

9. $\sqrt{121x^6}$

10. $\sqrt{(4k)^4}$

11. $\pm\sqrt{169r^4}$

12. $-\sqrt[3]{-27p^6}$

13. $-\sqrt{625y^2z^4}$

14. $\sqrt{36q^{34}}$

15. $\sqrt{100x^2y^4z^6}$

16. $\sqrt[3]{-0.027}$

17. $-\sqrt{-0.36}$

18. $\sqrt{0.64p^{10}}$

19. $\sqrt[4]{(2x)^8}$

20. $\sqrt{(11y^2)^4}$

21. $\sqrt[3]{(5a^2b)^6}$

22. $\sqrt{(3x-1)^2}$

23. $\sqrt[3]{(m-5)^6}$

24. $\sqrt{36x^2-12x+1}$

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6-4 Study Guide and Intervention *(continued)*

*n*th *Roots*

Approximate Radicals with a Calculator

Irrational Number	a number that cannot be expressed as a terminating or a repeating decimal

Radicals such as $\sqrt{2}$ and $\sqrt{3}$ are examples of irrational numbers. Decimal approximations for irrational numbers are often used in applications. These approximations can be easily found with a calculator.

Example **Use a calculator to approximate $\sqrt[5]{18.2}$ to three decimal places.**

$\sqrt[5]{18.2} \approx 1.787$

Exercises

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{62}$ **2.** $\sqrt{1050}$ **3.** $\sqrt[3]{0.054}$

4. $-\sqrt[4]{5.45}$ **5.** $\sqrt{5280}$ **6.** $\sqrt{18{,}600}$

7. $\sqrt{0.095}$ **8.** $\sqrt[3]{-15}$ **9.** $\sqrt[5]{100}$

10. $\sqrt[6]{856}$ **11.** $\sqrt{3200}$ **12.** $\sqrt{0.05}$

13. $\sqrt{12{,}500}$ **14.** $\sqrt{0.60}$ **15.** $-\sqrt[4]{500}$

16. $\sqrt[3]{0.15}$ **17.** $\sqrt[6]{4200}$ **18.** $\sqrt{75}$

19. LAW ENFORCEMENT The formula $r = 2\sqrt{5L}$ is used by police to estimate the speed r in miles per hour of a car if the length L of the car's skid mark is measures in feet. Estimate to the nearest tenth of a mile per hour the speed of a car that leaves a skid mark 300 feet long.

20. SPACE TRAVEL The distance to the horizon d miles from a satellite orbiting h miles above Earth can be approximated by $d = \sqrt{8000h + h^2}$. What is the distance to the horizon if a satellite is orbiting 150 miles above Earth?

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6-4 Skills Practice

*n*th Roots

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{230}$

2. $\sqrt{38}$

3. $-\sqrt{152}$

4. $\sqrt{5.6}$

5. $\sqrt[3]{88}$

6. $\sqrt[3]{-222}$

7. $-\sqrt[4]{0.34}$

8. $\sqrt[5]{500}$

Simplify.

9. $\pm\sqrt{81}$

10. $\sqrt{144}$

11. $\sqrt{(-5)^2}$

12. $\sqrt{-5^2}$

13. $\sqrt{0.36}$

14. $-\sqrt{\frac{4}{9}}$

15. $\sqrt[3]{-8}$

16. $-\sqrt[3]{27}$

17. $\sqrt[3]{0.064}$

18. $\sqrt[5]{32}$

19. $\sqrt[4]{81}$

20. $\sqrt{y^2}$

21. $\sqrt[3]{125c^3}$

22. $\sqrt{64x^6}$

23. $\sqrt[3]{-27a^6}$

24. $\sqrt{m^8p^4}$

25. $-\sqrt{100p^4t^2}$

26. $\sqrt[4]{16w^4v^8}$

27. $\sqrt{(-3c)^4}$

28. $\sqrt{(a+b)^2}$

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6-4 Practice

*n*th Roots

Simplify.

1. $\sqrt{0.81}$
2. $-\sqrt{324}$
3. $-\sqrt[4]{256}$
4. $\sqrt[6]{64}$
5. $\sqrt[3]{-64}$
6. $\sqrt[3]{0.512}$
7. $\sqrt[5]{-243}$
8. $-\sqrt[4]{1296}$
9. $\sqrt[5]{\frac{-1024}{243}}$
10. $\sqrt[5]{243x^{10}}$
11. $\sqrt{14a^2}$
12. $\sqrt{-(14a)^2}$
13. $\sqrt{49m^2t^8}$
14. $\sqrt{\frac{16m^2}{25}}$
15. $\sqrt[3]{-64r^2w^{15}}$
16. $\sqrt{(2x)^8}$
17. $-\sqrt[4]{625s^8}$
18. $\sqrt[3]{216p^3q^9}$
19. $\sqrt{676x^4y^6}$
20. $\sqrt[3]{-27x^9y^{12}}$
21. $-\sqrt{144m^8n^6}$
22. $\sqrt[5]{-32x^5y^{10}}$
23. $\sqrt[6]{(m+4)^6}$
24. $\sqrt[3]{(2x+1)^3}$
25. $-\sqrt{49a^{10}b^{16}}$
26. $\sqrt[4]{(x-5)^8}$
27. $\sqrt[3]{343d^6}$
28. $\sqrt{x^2+10x+25}$

Use a calculator to approximate each value to three decimal places.

29. $\sqrt{7.8}$
30. $-\sqrt{89}$
31. $\sqrt[3]{25}$
32. $\sqrt[3]{-4}$
33. $\sqrt[4]{1.1}$
34. $\sqrt[5]{-0.1}$
35. $\sqrt[6]{5555}$
36. $\sqrt[4]{(0.94)^2}$

37. **RADIANT TEMPERATURE** Thermal sensors measure an object's *radiant* temperature, which is the amount of energy radiated by the object. The *internal* temperature of an object is called its *kinetic* temperature. The formula $T_r = T_k\sqrt[4]{e}$ relates an object's radiant temperature T_r to its kinetic temperature T_k. The variable e in the formula is a measure of how well the object radiates energy. If an object's kinetic temperature is 30°C and $e = 0.94$, what is the object's radiant temperature to the nearest tenth of a degree?

38. **HERON'S FORMULA** Salvatore is buying fertilizer for his triangular garden. He knows the lengths of all three sides, so he is using Heron's formula to find the area. Heron's formula states that the area of a triangle is $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c are the lengths of the sides of the triangle and s is half the perimeter of the triangle. If the lengths of the sides of Salvatore's garden are 15 feet, 17 feet, and 20 feet, what is the area of the garden? Round your answer to the nearest whole number.

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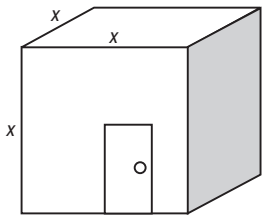
6-4 Word Problem Practice

*n*th Roots

1. CUBES Cathy is building a cubic storage room. She wants the volume of the space to be 1728 cubic feet. What should the dimensions of the cube be?



2. ASTRONOMY A special form of Kepler's Third Law of Planetary Motion is given by $a = \sqrt[3]{P^2}$, where a is the average distance of an object from the Sun in AU (astronomical units) and P is the period of the orbit in years. The period of Jupiter's orbit is 12 years. What is its distance from the Sun in AU?

3. TUNING Two notes are an octave apart if the frequency of the higher note is twice the frequency of the lower note. Casey is experimenting with an instrument that has 6 notes tuned so that the frequency of each successive note increases by the same factor and the first and last note are an octave apart. By what factor does the frequency increase from note to note?

4. MARKUPS A wholesaler manufactures a part for D dollars. The wholesaler sells the part to a dealer for a P percent markup. The dealer sells the part to a retailer at an additional P percent markup. The retailer in turn sells the part to its customers marking up the price yet another P percent. What is the price that customers see? If the customer buys the part for \$80 and the original cost to make the part was \$29.15, what is the markup?

5. PENDULUMS Mr. Topalian's physics class is experimenting with pendulums. The class learned the formula $T = 2\pi\sqrt{\frac{L}{g}}$ which relates the time T that it takes for a pendulum to swing back and forth based on gravity g, equal to 32 feet per second squared, and the length of the pendulum L in feet.

a. One group in the class made a 2-foot long pendulum. Use the formula to determine how long it will take for their pendulum to swing back and forth.

b. Another group decided they wanted to make a pendulum that took about 1.76 seconds to go back and forth. Approximately how long should their pendulum be?

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6-4 Enrichment

Approximating Square Roots

Consider the following expansion.

$$\left(a + \frac{b}{2a}\right)^2 = a^2 + \frac{2ab}{2a} + \frac{b^2}{4a^2}$$
$$= a^2 + b + \frac{b^2}{4a^2}$$

Think what happens if a is very great in comparison to b. The term $\frac{b^2}{4a^2}$ is very small and can be disregarded in an approximation.

$$\left(a + \frac{b}{2a}\right)^2 \approx a^2 + b$$
$$a + \frac{b}{2a} \approx \sqrt{a^2 + b}$$

Suppose a number can be expressed as $a^2 + b$, $a > b$. Then an approximate value of the square root is $a + \frac{b}{2a}$. You should also see that $a - \frac{b}{2a} \approx \sqrt{a^2 - b}$.

Example **Use the formula $\sqrt{a^2 \pm b} \approx a \pm \frac{b}{2a}$ to approximate $\sqrt{101}$ and $\sqrt{622}$.**

a. $\sqrt{101} = \sqrt{100 + 1} = \sqrt{10^2 + 1}$

Let $a = 10$ and $b = 1$.

$$\sqrt{101} \approx 10 + \frac{1}{2(10)}$$
$$\approx 10.05$$

b. $\sqrt{622} = \sqrt{625 - 3} = \sqrt{25^2 - 3}$

Let $a = 25$ and $b = 3$.

$$\sqrt{622} \approx 25 - \frac{3}{2(25)}$$
$$\approx 24.94$$

Exercises

Use the formula to find an approximation for each square root to the nearest hundredth. Check your work with a calculator.

1. $\sqrt{626}$ **2.** $\sqrt{99}$ **3.** $\sqrt{402}$

4. $\sqrt{1604}$ **5.** $\sqrt{223}$ **6.** $\sqrt{80}$

7. $\sqrt{4890}$ **8.** $\sqrt{2505}$ **9.** $\sqrt{3575}$

10. $\sqrt{1{,}441{,}100}$ **11.** $\sqrt{290}$ **12.** $\sqrt{260}$

13. Show that $a - \frac{b}{2a} \approx \sqrt{a^2 - b}$ for $a > b$.