NAME ____________________ DATE ____________ PERIOD ____________

6-2 Study Guide and Intervention

Inverse Functions and Relations

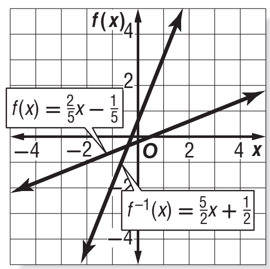
Find Inverses

Inverse Relations	Two relations are inverse relations if and only if whenever one relation contains the element (a, b), the other relation contains the element (b, a).
Property of Inverse Functions	Suppose f and f^{-1} are inverse functions. Then $f(a) = b$ if and only if $f^{-1}(b) = a$.

Example **Find the inverse of the function $f(x) = \frac{2}{5}x - \frac{1}{5}$. Then graph the function and its inverse.**

Step 1 Replace $f(x)$ with y in the original equation.

$$f(x) = \frac{2}{5}x - \frac{1}{5} \quad \rightarrow \quad y = \frac{2}{5}x - \frac{1}{5}$$



Step 2 Interchange x and y.

$$x = \frac{2}{5}y - \frac{1}{5}$$

Step 3 Solve for y.

$x = \frac{2}{5}y - \frac{1}{5}$	Inverse of $y = \frac{2}{5}x - \frac{1}{5}$
$5x = 2y - 1$	Multiply each side by 5.
$5x + 1 = 2y$	Add 1 to each side.
$\frac{1}{2}(5x + 1) = y$	Divide each side by 2.

The inverse of $f(x) = \frac{2}{5}x - \frac{1}{5}$ is $f^{-1}(x) = \frac{1}{2}(5x + 1)$.

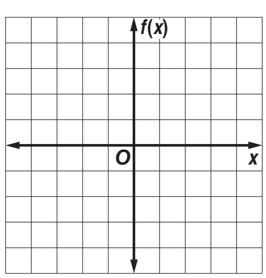
Exercises

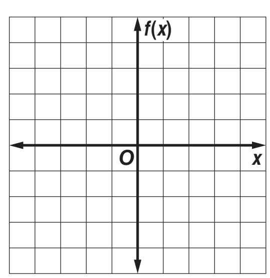
Find the inverse of each function. Then graph the function and its inverse.

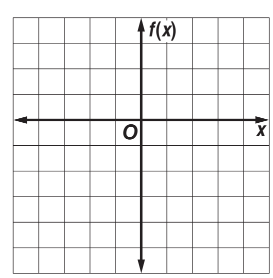
1. $f(x) = \frac{2}{3}x - 1$

2. $f(x) = 2x - 3$

3. $f(x) = \frac{1}{4}x - 2$







6-2 Study Guide and Intervention *(continued)*

Inverse Functions and Relations

Verifying Inverses

Inverse Functions	Two functions $f(x)$ and $g(x)$ are inverse functions if and only if $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 1 **Determine whether $f(x) = 2x - 7$ and $g(x) = \frac{1}{2}(x + 7)$ are inverse functions.**

$$[f \circ g](x) = f[g(x)] = f\left[\frac{1}{2}(x + 7)\right] = 2\left[\frac{1}{2}(x + 7)\right] - 7 = x + 7 - 7 = x$$

$$[g \circ f](x) = g[f(x)] = g(2x - 7) = \frac{1}{2}(2x - 7 + 7) = x$$

The functions are inverses since both $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 2 **Determine whether $f(x) = 4x + \frac{1}{3}$ and $g(x) = \frac{1}{4}x - 3$ are inverse functions.**

$$[f \circ g](x) = f[g(x)] = f\left(\frac{1}{4}x - 3\right) = 4\left(\frac{1}{4}x - 3\right) + \frac{1}{3} = x - 12 + \frac{1}{3} = x - 11\frac{2}{3}$$

Since $[f \circ g](x) \neq x$, the functions are not inverses.

Exercises

Determine whether each pair of functions are inverse functions. Write *yes* or *no*.

1. $f(x) = 3x - 1$, $g(x) = \frac{1}{3}x + \frac{1}{3}$

2. $f(x) = \frac{1}{4}x + 5$, $g(x) = 4x - 20$

3. $f(x) = \frac{1}{2}x - 10$, $g(x) = 2x + \frac{1}{10}$

4. $f(x) = 2x + 5$, $g(x) = 5x + 2$

5. $f(x) = 8x - 12$, $g(x) = \frac{1}{8}x + 12$

6. $f(x) = -2x + 3$, $g(x) = -\frac{1}{2}x + \frac{3}{2}$

7. $f(x) = 4x - \frac{1}{2}$, $g(x) = \frac{1}{4}x + \frac{1}{8}$

8. $f(x) = 2x - \frac{3}{5}$, $g(x) = \frac{1}{10}(5x + 3)$

9. $f(x) = 4x + \frac{1}{2}$, $g(x) = \frac{1}{2}x - \frac{3}{2}$

10. $f(x) = 10 - \frac{x}{2}$, $g(x) = 20 - 2x$

11. $f(x) = 4x - \frac{4}{5}$, $g(x) = \frac{x}{4} + \frac{1}{5}$

12. $f(x) = 9 + \frac{3}{2}x$, $g(x) = \frac{2}{3}x - 6$

Lesson 6-2

NAME ______________________ DATE ____________ PERIOD ____________

6-2 Skills Practice

Inverse Functions and Relations

Find the inverse of each relation.

1. $\{(3, 1), (4, -3), (8, -3)\}$

2. $\{(-7, 1), (0, 5), (5, -1)\}$

3. $\{(-10, -2), (-7, 6), (-4, -2), (-4, 0)\}$

4. $\{(0, -9), (5, -3), (6, 6), (8, -3)\}$

5. $\{(-4, 12), (0, 7), (9, -1), (10, -5)\}$

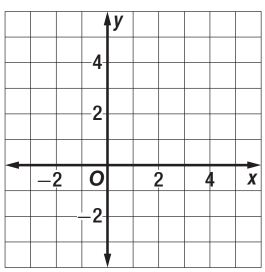
6. $\{(-4, 1), (-4, 3), (0, -8), (8, -9)\}$

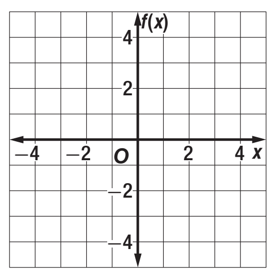
Find the inverse of each function. Then graph the function and its inverse.

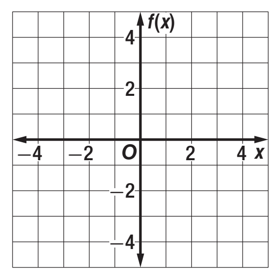
7. $y = 4$

8. $f(x) = 3x$

9. $f(x) = x + 2$



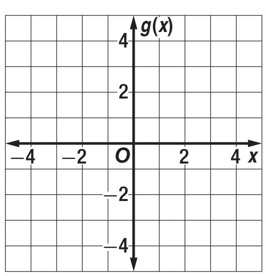


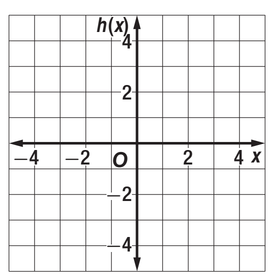


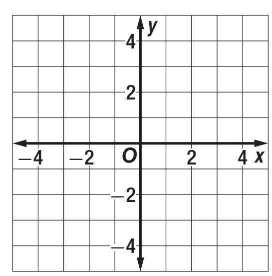
10. $g(x) = 2x - 1$

11. $h(x) = \frac{1}{4}x$

12. $y = \frac{2}{3}x + 2$







Determine whether each pair of functions are inverse functions. Write *yes* or *no*.

13. $f(x) = x - 1$
$g(x) = 1 - x$

14. $f(x) = 2x + 3$
$g(x) = \frac{1}{2}(x - 3)$

15. $f(x) = 5x - 5$
$g(x) = \frac{1}{5}x + 1$

16. $f(x) = 2x$
$g(x) = \frac{1}{2}x$

17. $h(x) = 6x - 2$
$g(x) = \frac{1}{6}x + 3$

18. $f(x) = 8x - 10$
$g(x) = \frac{1}{8}x + \frac{5}{4}$

6-2 Practice

Inverse Functions and Relations

Find the inverse of each relation.

1. {(0, 3), (4, 2), (5, −6)}

2. {(−5, 1), (−5, −1), (−5, 8)}

3. {(−3, −7), (0, −1), (5, 9), (7, 13)}

4. {(8, −2), (10, 5), (12, 6), (14, 7)}

5. {(−5, −4), (1, 2), (3, 4), (7, 8)}

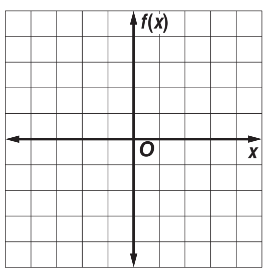
6. {(−3, 9), (−2, 4), (0, 0), (1, 1)}

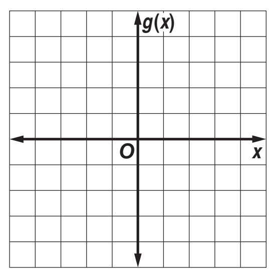
Find the inverse of each function. Then graph the function and its inverse.

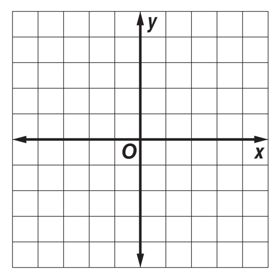
7. $f(x) = \frac{3}{4}x$

8. $g(x) = 3 + x$

9. $y = 3x - 2$







Determine whether each pair of functions are inverse functions. Write *yes* or *no*.

10. $f(x) = x + 6$
$g(x) = x - 6$

11. $f(x) = -4x + 1$
$g(x) = \frac{1}{4}(1 - x)$

12. $g(x) = 13x - 13$
$h(x) = \frac{1}{13}x - 1$

13. $f(x) = 2x$
$g(x) = -2x$

14. $f(x) = \frac{6}{7}x$
$g(x) = \frac{7}{6}x$

15. $g(x) = 2x - 8$
$h(x) = \frac{1}{2}x + 4$

16. MEASUREMENT The points (63, 121), (71, 180), (67, 140), (65, 108), and (72, 165) give the weight in pounds as a function of height in inches for 5 students in a class. Give the points for these students that represent height as a function of weight.

17. REMODELING The Clearys are replacing the flooring in their 15 foot by 18 foot kitchen. The new flooring costs $17.99 per square yard. The formula $f(x) = 9x$ converts square yards to square feet.

a. Find the inverse $f^{-1}(x)$. What is the significance of $f^{-1}(x)$ for the Clearys?

b. What will the new flooring cost the Clearys?

Lesson 6-2

6-2 Word Problem Practice

Inverse Functions and Relations

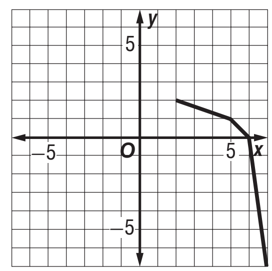
1. VOLUME Jason wants to make a spherical water cooler that can hold half a cubic meter of water. He knows that $V = \frac{4}{3}\pi r^3$, but he needs to know how to find r given V. Find this inverse function.

2. EXERCISE Alex began a new exercise routine. To gain the maximum benefit from his exercise, Alex calculated his maximum target heart rate using the function $f(x) = 0.85(220 - x)$, where x represents his age. Find the inverse of this function.

3. ROCKETS The altitude of a rocket in feet as a function of time is given by $f(t) = 49t^2$, where $t \geq 0$. Find the inverse of this function and determine the times when the rocket will be 10, 100, and 1000 feet high. Round your answers to the nearest hundredth of a second.

4. SELF-INVERTIBLE Karen finds the incomplete graph of a function in the back of her engineering handbook. The function is graphed in the figure below.

Karen knows that this function is its own inverse. Armed with this knowledge, extend the graph for values of x between -7 and 2.



5. PLANETS The approximate distance of a planet from the Sun is given by $d = T^{\frac{2}{3}}$, where d is distance in astronomical units and T is the period of its orbit in Earth years. An astronomical unit is the distance between Earth and the Sun.

a. Solve for T in terms of d.

b. Pluto is about 39.44 times as far from the Sun as Earth. About how many years does it take Pluto to orbit the Sun?

6-2 Enrichment

Reading Algebra

In mathematics, the term *group* has a special meaning. The following numbered sentences discuss the idea of group and one interesting example of a group.

01 To be a group, a set of elements and a binary operation must satisfy four conditions: the set must be closed under the operation, the operation must be associative, there must be an identity element, and every element must have an inverse.

02 The following six functions form a group under the operation of composition of functions: $f_1(x) = x$, $f_2(x) = \frac{1}{x}$, $f_3(x) = 1 - x$, $f_4(x) = \frac{(x-1)}{x}$, $f_5(x) = \frac{x}{(x-1)}$, and $f_6(x) = \frac{1}{(1-x)}$.

03 This group is an example of a noncommutative group. For example, $f_3 \circ f_2 = f_4$, but $f_2 \circ f_3 = f_6$.

04 Some experimentation with this group will show that the identity element is f_1.

05 Every element is its own inverse except for f_4 and f_6, each of which is the inverse of the other.

Use the paragraph to answer these questions.

1. Explain what it means to say that a set is *closed* under an operation. Is the set of positive integers closed under subtraction?

2. Subtraction is a noncommutative operation for the set of integers. Write an informal definition of noncommutative.

3. For the set of integers, what is the identity element for the operation of multiplication? Justify your answer.

4. Explain how the following statement relates to sentence 05:

$$(f_6 \cdot f_4)(x) = f_6[f_4(x)] = f_6\left(\frac{1}{(1-x)}\right) = \frac{1}{\frac{1-(x-1)}{x}} = x = f_1(x).$$

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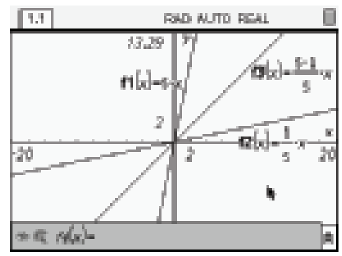
6-2 TI-Nspire® Activity

Inverse Functions and Relations

The **Graphs and Geometry** feature on the TI-Nspire calculator can be used to graph functions and their inverses in the same window.

Example **Graph $f(x) = 5x$ and its inverse, $f^{-1}(x) = \frac{1}{5}x$.**

Open a new **Graph** page from the Home button on the TI-Nspire.
Then enter the equation.



This graphs $f(x) = 5x$ on the screen. Next, graph $f^{-1}(x) = \frac{1}{5}x$ on the same screen.

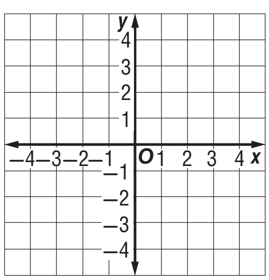
Test that the two functions are inverses by graphing $f[f^{-1}(x)]$.

The graph of $f[f^{-1}(x)]$ is identical to the graph of $y = x$, so the functions $f(x)$ and $f^{-1}(x)$ are inverses.

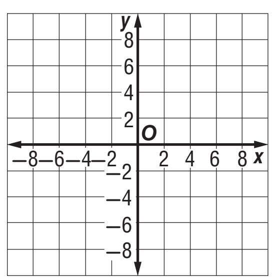
Exercises

Find the inverse of each function. Then graph the function and its inverse.

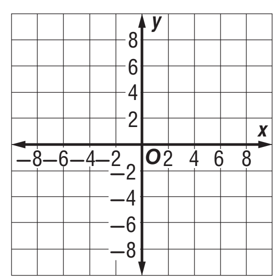
1. $f(x) = -2x + 2$



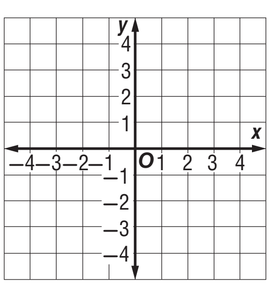
2. $f(x) = x + 5$



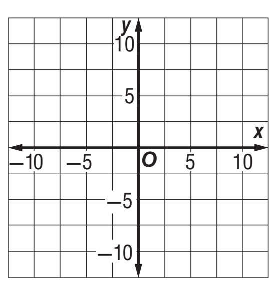
3. $f(x) = -\frac{1}{3}x + 2$



4. $f(x) = 3x - 6$



5. $f(x) = \frac{2}{3}x - 2$



6. $f(x) = -\frac{4}{3}x + 8$