# 6-1 Study Guide and Intervention

## *Operations on Functions*

## Arithmetic Operations

### Operations with Functions

| | |
|---|---|
| Sum | $(f + g)(x) = f(x) + g(x)$ |
| Difference | $(f - g)(x) = f(x) - g(x)$ |
| Product | $(f \cdot g)(x) = f(x) \cdot g(x)$ |
| Quotient | $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$ |



**Example** **Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for $f(x) = x^2 + 3x - 4$ and $g(x) = 3x - 2$.**

$(f + g)(x) = f(x) + g(x)$ Addition of functions
$= (x^2 + 3x - 4) + (3x - 2)$ $f(x) = x^2 + 3x - 4$, $g(x) = 3x - 2$
$= x^2 + 6x - 6$ Simplify.

$(f - g)(x) = f(x) - g(x)$ Subtraction of functions
$= (x^2 + 3x - 4) - (3x - 2)$ $f(x) = x^2 + 3x - 4$, $g(x) = 3x - 2$
$= x^2 - 2$ Simplify.

$(f \cdot g)(x) = f(x) \cdot g(x)$ Multiplication of functions
$= (x^2 + 3x - 4)(3x - 2)$ $f(x) = x^2 + 3x - 4$, $g(x) = 3x - 2$
$= x^2(3x - 2) + 3x(3x - 2) - 4(3x - 2)$ Distributive Property
$= 3x^3 - 2x^2 + 9x^2 - 6x - 12x + 8$ Distributive Property
$= 3x^3 + 7x^2 - 18x + 8$ Simplify.

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ Division of functions
$= \frac{x^2 + 3x - 4}{3x - 2}, x \neq \frac{2}{3}$ $f(x) = x^2 + 3x - 4$ and $g(x) = 3x - 2$

## Exercises

**Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.**

**1.** $f(x) = 8x - 3$; $g(x) = 4x + 5$

**2.** $f(x) = x^2 + x - 6$; $g(x) = x - 2$

**3.** $f(x) = 3x^2 - x + 5$; $g(x) = 2x - 3$

**4.** $f(x) = 2x - 1$; $g(x) = 3x^2 + 11x - 4$

**5.** $f(x) = x^2 - 1$; $g(x) = \frac{1}{x + 1}$

# 6-1 Study Guide and Intervention *(continued)*

## *Operations on Functions*

**Composition of Functions** Suppose $f$ and $g$ are functions such that the range of $g$ is a subset of the domain of $f$. Then the composite function $f \circ g$ can be described by the equation $[f \circ g](x) = f[g(x)]$.

**Example 1** **For $f = \{(1, 2), (3, 3), (2, 4), (4, 1)\}$ and $g = \{(1, 3), (3, 4), (2, 2), (4, 1)\}$, find $f \circ g$ and $g \circ f$ if they exist.**

$f[g(1)] = f(3) = 3$ $f[g(2)] = f(2) = 4$ $f[g(3)] = f(4) = 1$ $f[g(4)] = f(1) = 2$,

So $f \circ g = \{(1, 3), (2, 4), (3, 1), (4, 2)\}$

$g[f(1)] = g(2) = 2$ $g[f(2)] = g(4) = 1$ $g[f(3)] = g(3) = 4$ $g[f(4)] = g(1) = 3$,

So $g \circ f = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$

**Example 2** **Find $[g \circ h](x)$ and $[h \circ g](x)$ for $g(x) = 3x - 4$ and $h(x) = x^2 - 1$.**

$$\begin{aligned}[g \circ h](x) &= g[h(x)] \\ &= g(x^2 - 1) \\ &= 3(x^2 - 1) - 4 \\ &= 3x^2 - 7\end{aligned}$$

$$\begin{aligned}[h \circ g](x) &= h[g(x)] \\ &= h(3x - 4) \\ &= (3x - 4)^2 - 1 \\ &= 9x^2 - 24x + 16 - 1 \\ &= 9x^2 - 24x + 15\end{aligned}$$

## Exercises

**For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist.**

**1.** $f = \{(-1, 2), (5, 6), (0, 9)\}$,
$g = \{(6, 0), (2, -1), (9, 5)\}$

**2.** $f = \{(5, -2), (9, 8), (-4, 3), (0, 4)\}$,
$g = \{(3, 7), (-2, 6), (4, -2), (8, 10)\}$

**Find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist.**

**3.** $f(x) = 2x + 7$; $g(x) = -5x - 1$

**4.** $f(x) = x^2 - 1$; $g(x) = -4x^2$

**5.** $f(x) = x^2 + 2x$; $g(x) = x - 9$

**6.** $f(x) = 5x + 4$; $g(x) = 3 - x$

# 6-1 Skills Practice

## *Operations on Functions*



**Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.**

**1.** $f(x) = x + 5$
$g(x) = x - 4$

**2.** $f(x) = 3x + 1$
$g(x) = 2x - 3$

**3.** $f(x) = x^2$
$g(x) = 4 - x$

**4.** $f(x) = 3x^2$
$g(x) = \frac{5}{x}$

**For each pair of functions, find $f \circ g$ and $g \circ f$ if they exist.**

**5.** $f = \{(0, 0), (4, -2)\}$
$g = \{(0, 4), (-2, 0), (5, 0)\}$

**6.** $f = \{(0, -3), (1, 2), (2, 2)\}$
$g = \{(-3, 1), (2, 0)\}$

**7.** $f = \{(-4, 3), (-1, 1), (2, 2)\}$
$g = \{(1, -4), (2, -1), (3, -1)\}$

**8.** $f = \{(6, 6), (-3, -3), (1, 3)\}$
$g = \{(-3, 6), (3, 6), (6, -3)\}$

**Find $[g \circ h](x)$ and $[h \circ g](x)$ if they exist.**

**9.** $g(x) = 2x$
$h(x) = x + 2$

**10.** $g(x) = -3x$
$h(x) = 4x - 1$

**11.** $g(x) = x - 6$
$h(x) = x + 6$

**12.** $g(x) = x - 3$
$h(x) = x^2$

**13.** $g(x) = 5x$
$h(x) = x^2 + x - 1$

**14.** $g(x) = x + 2$
$h(x) = 2x^2 - 3$

**If $f(x) = 3x$, $g(x) = x + 4$, and $h(x) = x^2 - 1$, find each value.**

**15.** $f[g(1)]$

**16.** $g[h(0)]$

**17.** $g[f(-1)]$

**18.** $h[f(5)]$

**19.** $g[h(-3)]$

**20.** $h[f(10)]$

**21.** $f[h(8)]$

**22.** $[f \circ (h \circ g)](1)$

**23.** $[f \circ (g \circ h)](-2)$

# 6-1 Practice

## *Operations on Functions*

**Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.**

**1.** $f(x) = 2x + 1$
$g(x) = x - 3$

**2.** $f(x) = 8x^2$
$g(x) = \frac{1}{x^2}$

**3.** $f(x) = x^2 + 7x + 12$
$g(x) = x^2 - 9$

**For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist.**

**4.** $f = \{(-9, -1), (-1, 0), (3, 4)\}$
$g = \{(0, -9), (-1, 3), (4, -1)\}$

**5.** $f = \{(-4, 3), (0, -2), (1, -2)\}$
$g = \{(-2, 0), (3, 1)\}$

**6.** $f = \{(-4, -5), (0, 3), (1, 6)\}$
$g = \{(6, 1), (-5, 0), (3, -4)\}$

**7.** $f = \{(0, -3), (1, -3), (6, 8)\}$
$g = \{(8, 2), (-3, 0), (-3, 1)\}$

**Find $[g \circ h](x)$ and $[h \circ g](x)$, if they exist.**

**8.** $g(x) = 3x$
$h(x) = x - 4$

**9.** $g(x) = -8x$
$h(x) = 2x + 3$

**10.** $g(x) = x + 6$
$h(x) = 3x^2$

**11.** $g(x) = x + 3$
$h(x) = 2x^2$

**12.** $g(x) = -2x$
$h(x) = x^2 + 3x + 2$

**13.** $g(x) = x - 2$
$h(x) = 3x^2 + 1$

**If $f(x) = x^2$, $g(x) = 5x$, and $h(x) = x + 4$, find each value.**

**14.** $f[g(1)]$

**15.** $g[h(-2)]$

**16.** $h[f(4)]$

**17.** $f[h(-9)]$

**18.** $h[g(-3)]$

**19.** $g[f(8)]$

**20. BUSINESS** The function $f(x) = 1000 - 0.01x^2$ models the manufacturing cost per item when $x$ items are produced, and $g(x) = 150 - 0.001x^2$ models the service cost per item. Write a function $C(x)$ for the total manufacturing and service cost per item.

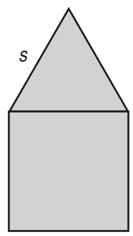
**21. MEASUREMENT** The formula $f = \frac{n}{12}$ converts inches $n$ to feet $f$, and $m = \frac{f}{5280}$ converts feet to miles $m$. Write a composition of functions that converts inches to miles.

# 6-1 Word Problem Practice

## Operations on Functions



**1. AREA** Bernard wants to know the area of a figure made by joining an equilateral triangle and square along an edge. The function $f(s) = \frac{\sqrt{3}}{4}s^2$ gives the area of an equilateral triangle with side $s$. The function $g(s) = s^2$ gives the area of a square with side $s$. What function $h(s)$ gives the area of the figure as a function of its side length $s$?

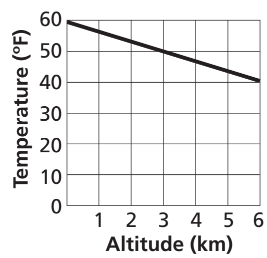


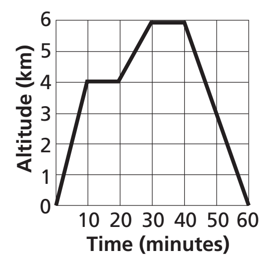

**2. PRICING** A computer company decides to continuously adjust the pricing of and discounts to its products in an effort to remain competitive. The function $P(t)$ gives the sale price of its Super2000 computer as a function of time. The function $D(t)$ gives the value of a special discount it offers to valued customers. How much would valued customers have to pay for one Super2000 computer?

**3. LAVA** The temperature of lava has been measured at up to 2000°F. A freshly ejected lava rock immediately begins to cool down. The temperature of the lava rock in degrees Fahrenheit as a function of time is given by $T(t)$. Let $C(F)$ be the function that gives degrees Celsius as a function of degrees Fahrenheit. What function gives the temperature of the lava rock in degrees Celsius as a function of time?

**4. ENGINEERING** A group of engineers is designing a staple gun. One team determines that the speed of impact $s$ of the staple (in feet per second) as a function of the handle length $\ell$ (in inches) is given by $s(\ell) = 40 + 3\ell$. A second team determines that the number of sheets $N$ that can be stapled as a function of the impact speed is given by $N(s) = \frac{s - 10}{3}$. What function gives $N$ as a function of $\ell$?

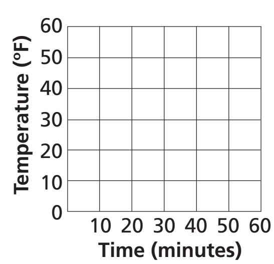
**5. HOT AIR BALLOONS** Hannah and Terry went on a one-hour hot air balloon ride. Let $T(A)$ be the outside air temperature as a function of altitude and let $A(t)$ be the altitude of the balloon as a function of time.




**a.** What function describes the air temperature Hannah and Terry felt at different times during their trip?

**b.** Sketch a graph of the function you wrote for part **a** based on the graphs for $T(A)$ and $A(t)$ that are given.

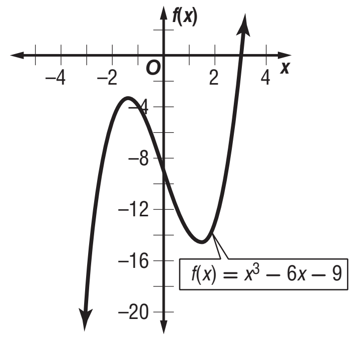


# 6-1 Enrichment

## *Relative Maximum Values*

The graph of $f(x) = x^3 - 6x - 9$ shows a relative maximum value somewhere between $f(-2)$ and $f(-1)$. You can obtain a closer approximation by comparing values such as those shown in the table.

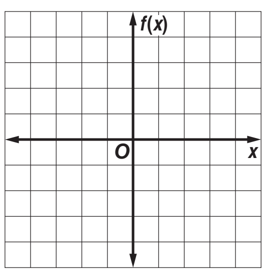
To the nearest tenth a relative maximum value for $f(x)$ is $-3.3$.



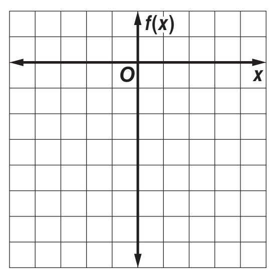

| x | f(x) |
|---|---|
| −2 | −5 |
| −1.5 | −3.375 |
| −1.4 | −3.344 |
| −1.3 | −3.397 |
| −1 | −4 |

**Using a calculator to find points, graph each function. To the nearest tenth, find a relative maximum value of the function.**

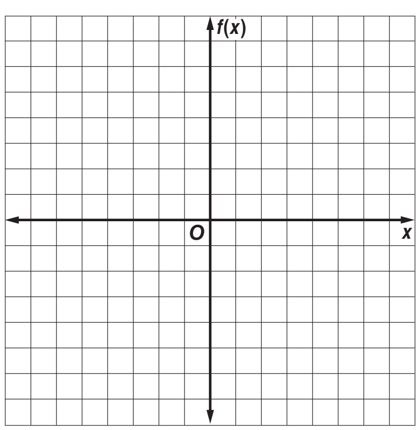
**1.** $f(x) = x(x^2 - 3)$



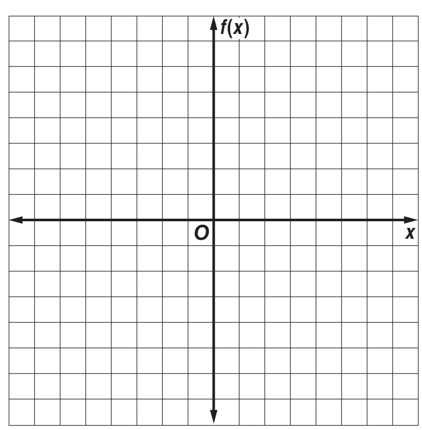

**2.** $f(x) = x^3 - 3x - 3$




**3.** $f(x) = x^3 - 9x - 2$




**4.** $f(x) = x^3 + 2x^2 - 12x - 24$



NAME ______________________ DATE ____________ PERIOD ____________

# 6-1 Spreadsheet Activity

## *Operations on Functions*

It is possible to perform operations on functions such as addition, subtraction, multiplication and division. You can use a spreadsheet to investigate the relationships among functions.

**Consider the functions $f(x) = 3x + 2$, $g(x) = x^2 - 2x$, and $h(x) = x^2 + x + 2$. Find the function values of each function for several values of $x$. Does it appear that $f(x) + g(x) = h(x)$?**

Use Column A for the chosen values of $x$. Columns B, C, and E are $f(x)$, $g(x)$, and $h(x)$ respectively. Use Column D for $f(x) + g(x)$.

For every value of $x$, $f(x) + g(x) = h(x)$.

**Functions.xls**

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | x | f(x) | g(x) | f(x) + g(x) | h(x) |
| 2 | –4 | –10 | 24 | 14 | 14 |
| 3 | –2.5 | –5.5 | 11.25 | 5.75 | 5.75 |
| 4 | –1 | –1 | 3 | 2 | 2 |
| 5 | 0 | 2 | 0 | 2 | 2 |
| 6 | 1 | 5 | –1 | 4 | 4 |
| 7 | 4 | 14 | 8 | 22 | 22 |
| 8 | 12 | 38 | 120 | 158 | 158 |
| 9 | | | | | |

Sheet 1 | Sheet 2 | Sheet 3

Lesson 6-1

## Exercises

**Study and use the spreadsheet above.**

**1.** Find $k(x) = (3x + 2) + (x^2 - 2x)$. How does it compare to $h(x)$?

**2.** Change the functions in the spreadsheet to $f(x) = \frac{x}{2}$, $g(x) = 1 - x^2$, and $h(x) = 1 + \frac{x}{2} - x^2$. How are these functions related? Is it true that $f(x) + g(x) = h(x)$?

**3.** Make a conjecture about $(f + g)(x)$ for any functions $f(x)$ and $g(x)$.

**4.** Make a conjecture about $(f - g)(x)$ for any functions $f(x)$ and $g(x)$. Use the spreadsheet to test your conjecture. Does it appear to be true? Explain your answer.

**Find $(f + g)(x)$, $(f - g)(x)$, for each $f(x)$ and $g(x)$. Use the spreadsheet to find function values to verify your solutions.**

**5.** $f(x) = 6x + 8$
$g(x) = 9 + x$

**6.** $f(x) = x^2 + 1$
$g(x) = 3x - 4$

**7.** $f(x) = 10x^2$
$g(x) = 6 - x^2$