# 3-7 Study Guide and Intervention

## *Solving Systems of Equations Using Cramer's Rule*

**Determinants** A 2×2 matrix has a second-order determinant; a 3×3 matrix has a third-order determinant.

| | |
|---|---|
| **Second-Order Determinant** | For the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$. |
| **Third-Order Determinant** | For the matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, the determinant is found using the diagonal rule. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{matrix} a & b \\ d & e \\ g & h \end{matrix}$ $\quad$ $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{matrix} a & b \\ d & e \\ g & h \end{matrix}$ |
| **Area of a Triangle** | The area of a triangle having vertices $(a, b)$, $(c, d)$, and $(e, f)$ is $\lvert A \rvert$, where $A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$. |

**Example** **Evaluate each determinant.**

**a.** $\begin{vmatrix} 6 & 3 \\ -8 & 5 \end{vmatrix}$

$$\begin{vmatrix} 6 & 3 \\ -8 & 5 \end{vmatrix} = 6\,(5) - 3\,(-8)$$
$$= 54$$

**b.** $\begin{vmatrix} 4 & 5 & -2 \\ 1 & 3 & 0 \\ 2 & -3 & 6 \end{vmatrix}$

$$\begin{vmatrix} 4 & 5 & -2 \\ 1 & 3 & 0 \\ 2 & -3 & 6 \end{vmatrix}\begin{matrix} 4 & 5 \\ 1 & 3 \\ 2 & -3 \end{matrix} \qquad \begin{vmatrix} 4 & 5 & -2 \\ 1 & 3 & 0 \\ 2 & -3 & 6 \end{vmatrix}\begin{matrix} 4 & 5 \\ 1 & 3 \\ 2 & -3 \end{matrix}$$

$= [4(3)6 + 5(0)2 + (-2)1(-3)] - [(-2)3(2) + 4(0)(-3) + 5(1)6]$

$= [72 + 0 + 6] - [-12 + 0 + 30]$

$= 78 - 16$ or $60$

## Exercises

**Evaluate each determinant.**

**1.** $\begin{vmatrix} 6 & -2 \\ 5 & 7 \end{vmatrix}$

**2.** $\begin{vmatrix} 3 & 2 \\ 9 & 6 \end{vmatrix}$

**3.** $\begin{vmatrix} 3 & -2 & -2 \\ 0 & 4 & 1 \\ -1 & 4 & -3 \end{vmatrix}$

**4.** Find the area of a triangle with vertices (2, –3), (7, 4), and (–5, 5).

Lesson 3-7

# 3-7 Study Guide and Intervention *(continued)*

## *Solving Systems of Equations Using Cramer's Rule*

**Cramer's Rule** Determinants provide a way for solving systems of equations.

| | |
|---|---|
| **Cramer's Rule for Two-Variable Systems** | Let $C$ be the coefficient matrix of the system $\begin{matrix} ax + by = m \\ fx + gy = n \end{matrix}$ $\rightarrow$ $\begin{vmatrix} a & b \\ f & g \end{vmatrix}$<br>The solution of this system is $x = \dfrac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{\lvert C \rvert}$, $y = \dfrac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{\lvert C \rvert}$, if $C \neq 0$. |

**Example** **Use Cramer's Rule to solve the system of equations.** $\mathbf{5x - 10y = 8}$
$\mathbf{10x + 25y = -2}$

$x = \dfrac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{\lvert C \rvert}$ Cramer's Rule $y = \dfrac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{\lvert C \rvert}$

$= \dfrac{\begin{vmatrix} 8 & -10 \\ -2 & 25 \end{vmatrix}}{\begin{vmatrix} 5 & -10 \\ 10 & 25 \end{vmatrix}}$ $a = 5, b = -10, f = 10, g = 25, m = 8, n = -2$ $= \dfrac{\begin{vmatrix} 5 & 8 \\ 10 & -2 \end{vmatrix}}{\begin{vmatrix} 5 & -10 \\ 10 & 25 \end{vmatrix}}$

$= \dfrac{8(25) - (-2)(-10)}{5(25) - (-10)(10)}$ Evaluate each determinant. $= \dfrac{5(-2) - 8(10)}{5(25) - (-10)(10)}$

$= \dfrac{180}{225}$ or $\dfrac{4}{5}$ Simplify. $= -\dfrac{90}{225}$ or $-\dfrac{2}{5}$

The solution is $\left(\dfrac{4}{5}, -\dfrac{2}{5}\right)$.

## Exercises

**Use Cramer's Rule to solve each system of equations.**

**1.** $3x - 2y = 7$
$2x + 7y = 38$

**2.** $x - 4y = 17$
$3x - y = 29$

**3.** $2x - y = -2$
$4x - y = 4$

**4.** $2x - y = 1$
$5x + 2y = -29$

**5.** $4x + 2y = 1$
$5x - 4y = 24$

**6.** $6x - 3y = -3$
$2x + y = 21$

**7.** $2x + 7y = 16$
$x - 2y = 30$

**8.** $2x - 3y = -2$
$3x - 4y = 9$

**9.** $\dfrac{x}{3} + \dfrac{y}{5} = 2$
$\dfrac{x}{4} - \dfrac{y}{6} = -8$

**10.** $6x - 9y = -1$
$3x + 18y = 12$

**11.** $3x - 12y = -14$
$9x + 6y = -7$

**12.** $8x + 2y = \dfrac{3}{7}$
$5x - 4y = -\dfrac{27}{7}$

# 3-7 Skills Practice

## *Solving Systems of Equations Using Cramer's Rule*

**Evaluate each determinant.**

**1.** $\begin{vmatrix} 5 & 2 \\ 1 & 3 \end{vmatrix}$

**2.** $\begin{vmatrix} 10 & 9 \\ 5 & 8 \end{vmatrix}$

**3.** $\begin{vmatrix} 1 & 6 \\ 1 & 7 \end{vmatrix}$

**4.** $\begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix}$

**5.** $\begin{vmatrix} 0 & 9 \\ 5 & 8 \end{vmatrix}$

**6.** $\begin{vmatrix} 3 & 12 \\ 2 & 8 \end{vmatrix}$

**7.** $\begin{vmatrix} -5 & 2 \\ 8 & -6 \end{vmatrix}$

**8.** $\begin{vmatrix} -3 & 1 \\ 8 & -7 \end{vmatrix}$

**9.** $\begin{vmatrix} 9 & -2 \\ -4 & 1 \end{vmatrix}$

**10.** $\begin{vmatrix} 1 & -5 \\ 1 & 6 \end{vmatrix}$

**11.** $\begin{vmatrix} 1 & -3 \\ -3 & 4 \end{vmatrix}$

**12.** $\begin{vmatrix} -12 & 4 \\ 1 & 4 \end{vmatrix}$

**13.** $\begin{vmatrix} 3 & -5 \\ 6 & -11 \end{vmatrix}$

**14.** $\begin{vmatrix} -1 & -3 \\ 5 & -2 \end{vmatrix}$

**15.** $\begin{vmatrix} -1 & -14 \\ 5 & 2 \end{vmatrix}$

**16.** $\begin{vmatrix} -1 & 2 \\ 0 & 4 \end{vmatrix}$

**17.** $\begin{vmatrix} 2 & 2 \\ -1 & 4 \end{vmatrix}$

**18.** $\begin{vmatrix} -1 & 6 \\ 2 & 5 \end{vmatrix}$

**Evaluate each determinant using diagonals.**

**19.** $\begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 2 & 3 & -2 \end{vmatrix}$

**20.** $\begin{vmatrix} 6 & -1 & 1 \\ 5 & 2 & -1 \\ 1 & 3 & -2 \end{vmatrix}$

**21.** $\begin{vmatrix} 2 & 6 & 1 \\ 3 & 5 & -1 \\ 2 & 1 & -2 \end{vmatrix}$

**22.** $\begin{vmatrix} 2 & -1 & 6 \\ 3 & 2 & 5 \\ 2 & 3 & 1 \end{vmatrix}$

**23.** $\begin{vmatrix} 3 & -1 & 2 \\ 1 & 0 & 4 \\ 3 & -2 & 0 \end{vmatrix}$

**24.** $\begin{vmatrix} 3 & 2 & 2 \\ 1 & -1 & 4 \\ 3 & -1 & 0 \end{vmatrix}$

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# 3-7 Practice

## *Solving Systems of Equations Using Cramer's Rule*

**Evaluate each determinant.**

**1.** $\begin{vmatrix} 1 & 6 \\ 2 & 7 \end{vmatrix}$

**2.** $\begin{vmatrix} 9 & 6 \\ 3 & 2 \end{vmatrix}$

**3.** $\begin{vmatrix} 4 & 1 \\ -2 & -5 \end{vmatrix}$

**4.** $\begin{vmatrix} -14 & -3 \\ 2 & -2 \end{vmatrix}$

**5.** $\begin{vmatrix} 4 & -3 \\ -12 & 4 \end{vmatrix}$

**6.** $\begin{vmatrix} 2 & -5 \\ 5 & -11 \end{vmatrix}$

**7.** $\begin{vmatrix} 3 & -4 \\ 3.75 & 5 \end{vmatrix}$

**8.** $\begin{vmatrix} 2 & -1 \\ 3 & -9.5 \end{vmatrix}$

**9.** $\begin{vmatrix} 0.5 & -0.7 \\ 0.4 & -0.3 \end{vmatrix}$

**Evaluate each determinant using expansion by diagonals.**

**10.** $\begin{vmatrix} -2 & 3 & 1 \\ 0 & 4 & -3 \\ 2 & 5 & -1 \end{vmatrix}$

**11.** $\begin{vmatrix} 2 & -4 & 1 \\ 3 & 0 & 9 \\ -1 & 5 & 7 \end{vmatrix}$

**12.** $\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -2 \\ 1 & 1 & -1 \end{vmatrix}$

**13.** $\begin{vmatrix} 0 & -4 & 0 \\ 2 & -1 & 1 \\ 3 & -2 & 5 \end{vmatrix}$

**14.** $\begin{vmatrix} 2 & 7 & -6 \\ 8 & 4 & 0 \\ 1 & -1 & 3 \end{vmatrix}$

**15.** $\begin{vmatrix} -12 & 0 & 3 \\ 7 & 5 & -1 \\ 4 & 2 & -6 \end{vmatrix}$

**Use Cramer's Rule to solve each system of equation.**

**16.** $4x - 2y = -6$
$3x + y = 18$

**17.** $5x + 4y = 10$
$-3x - 2y = -8$

**18.** $-2x - 3y = -14$
$4x - y = 0$

**19.** $6x + 6y = 9$
$4x - 4y = -42$

**20.** $5x - 6 = 3y$
$5y = 54 + 3x$

**21.** $\frac{x}{2} + \frac{y}{4} = 2$
$\frac{x}{4} - \frac{y}{6} = -6$

**25. GEOMETRY** Find the area of a triangle whose vertices have coordinates (3, 5), (6, −5), and (−4, 10).

**26. LAND MANAGEMENT** A fish and wildlife management organization uses a GIS (geographic information system) to store and analyze data for the parcels of land it manages. All of the parcels are mapped on a grid in which 1 unit represents 1 acre. If the coordinates of the corners of a parcel are (−8, 10), (6, 17), and (2, −4), how many acres is the parcel?

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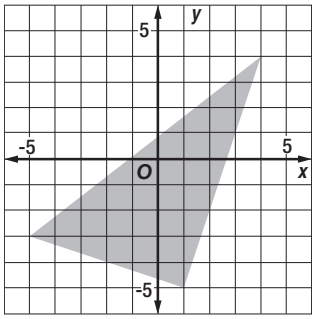
# 3-7 Word Problem Practice

## *Solve Systems of Equations Using Cramer's Rule*

**1. FIND THE ERROR** Mark's determinant computation has sign errors. Circle the signs that must be reversed.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1(5)(9) - 2(6)(7) + 3(4)(8) - 3(5)(7) + 1(6)(8) - 2(4)(9)$$

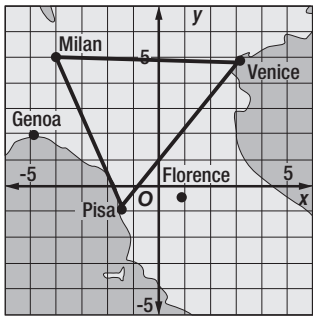
**2. POOL** An architect has a pool in the floor plans for a home. Set up a determinant that gives the unit area of the pool.




**3. HALF-UNIT TRIANGLES** For a school art project, students had to decorate a pegboard by looping strings around the pegs. Ronald wanted to make triangles with areas of one half square unit. Because Ronald had studied determinants, he knew that this was essentially the same as finding the coordinates of the vertices of a triangle $(a, b)$, $(c, d)$ and $(e, f)$, so that the determinant $\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$ is 1 or $-1$.

Give an example of such a triangle.

**4. ITALY** The figure shows a map of Italy overlaid on a graph. The coordinates of Milan, Venice, and Pisa are about $(-4, 5)$, $(3.25, 4.8)$, and $(-1.4, -0.8)$, respectively. Each square unit on the map represents about 400 square miles.




What is the area of the triangular region? Round your answer to the nearest square mile.

**5. ARROWS** Kyle is making a triangle with vertices at $(-6, 0)$, $(0, -x)$, and $(0, x)$, and $x > 0$. He plans to make the triangle using a material that costs \$2 for every square unit.

**a.** Write the determinant that gives the area of this triangle.

**b.** Evaluate the determinant you wrote for part **a** and determine the value of $x$ that results in a \$60 triangle.

Lesson 3-7

# 3-7 Enrichment

## *Matrix Transpose and Determinants*

In Lesson 4-1, you learned how to represent information in matrices. A matrix contains elements of the form $a_{ij}$ where $i$ is the row number of the element and $j$ is the columns number of the element.

Consider the following matrix.

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

In this matrix, $a_{11} = 2$, $a_{12} = -1$, $a_{21} = 3$, and $a_{22} = 4$.

The matrix transpose can be found by switching the elements around. Element $a_{ij}$ becomes element $a_{ji}$. So, the matrix transpose of $A$, denoted by $A^T$ is:

$$A^T = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$$

Calculate the determinant of $A$ and $A^T$.

$\det(A) = 2(4) - 3(-1) = 11$

$\det(A^T) = 2(4) - (-1)(3) = 11$

**1.** Find each matrix transpose.

**a.** $B = \begin{bmatrix} -1 & 5 \\ 2 & 6 \end{bmatrix}$ **b.** $C = \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$ **c.** $D = \begin{bmatrix} 2 & 3 & -1 \\ -2 & -1 & 5 \\ 1 & 3 & -2 \end{bmatrix}$

**2.** Find the determinants of the original matrices and the transposes from Exercise 1.

**3.** What do you notice about the determinants? Make a conjecture about the determinant of a matrix and the determinant of its transpose.

# 3-7 Spreadsheet Activity

## *Cramer's Rule*

You have learned to solve systems of linear equations by using matrix equations and the inverse matrix. Another way to solve systems is to use *Cramer's Rule*. Study the spreadsheet below to discover Cramer's Rule.

**Cramer's Rule.xls**

| | A | B | C |
|---|---|---|---|
| 1 | 6 | 3 | -12 |
| 2 | 5 | 1 | 8 |
| 3 | | | |
| 4 | =A1*B2-B1*A2 | | |
| 5 | | | |
| 6 | =C1*B2-B1*C2 | | |
| 7 | | | |
| 8 | =A1*C2-C1*A2 | | |
| 9 | | | |
| 10 | x = | =(A6/A4) | |
| 11 | y = | =(A8/A4) | |

Sheet 1 Sheet 2 Sheet 3

To use the spreadsheet to solve a system of equations, write each equation in the form below.

$$ax + by = c$$

The values for the system $6x + 3y = -12$ and $5x + y = 8$ are shown. In the spreadsheet, the values of $a$, $b$, and $c$ for the first equation are entered in cells A1, B1, and C1, respectively. The values of $a$, $b$, and $c$ for the second equation are entered in cells A2, B2, and C2, respectively.

The values in cells B10 and B11 represent the solution for the system.

## Exercises

**1.** Study the formula in cell A4. Write a matrix whose determinant is found using this formula.

**2.** Write matrices whose determinants are found using the formulas in cells A6 and A8.

**3.** Explain how the values of $x$ and $y$ are found using Cramer's Rule.

**Use the spreadsheet to solve each system of equations.**

**4.** $6x + 3y = -12$
$5x + y = 8$

**5.** $5x - 3y = 19$
$7x + 2y = 8$

**6.** $8x - 3y = 11$
$6x + 9y = 15$

**7.** $0.3x + 1.6y = 0.44$
$0.4x + 2.5y = 0.66$

**8.** $3y = 4x + 28$
$5x + 7y = 8$

**9.** $y = -0.5x + 4$
$y = 4x - 5$