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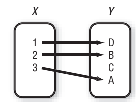
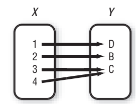
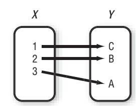
2-1 Study Guide and Intervention

Relations and Functions

Relations and Functions A **relation** can be represented as a set of ordered pairs or as an equation; the relation is then the set of all ordered pairs (x, y) that make the equation true. A **function** is a relation in which each element of the domain is paired with exactly one element of the range.

One-to-One Function	Each element of the domain pairs to exactly one **unique** element of the range.	
Onto Function	Each element of the range also corresponds to an element of the domain.	
Both One-to-One and Onto	Each element of the domain is paired to exactly one element of the range and each element of the range.	

Lesson 2-1

Example **State the domain and range of the relation. Does the relation represent a function?**

The domain and range are both all real numbers. Each element of the domain corresponds with exactly one element of the range, so it is a function.

x	y
–1	–5
0	–3
1	–1
2	1
3	3

Exercises

State the domain and range of each relation. Then determine whether each relation is a *function*. If it is a function, determine if it is *one-to-one, onto, both,* or *neither*.

1. {(0.5, 3), (0.4, 2), (3.1, 1), (0.4, 0)}

2. {(–5, 2), (4, –2), (3, –11), (–7, 2)}

3. {(0.5, –3), (0.1, 12), (6, 8)}

4. {(–15, 12), (–14, 11), (–13, 10), (–12, 12)}

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2-1 Study Guide and Intervention *(continued)*

Relations and Functions

Equations of Relations and Functions Equations that represent functions are often written in **functional notation**. For example, $y = 10 - 8x$ can be written as $f(x) = 10 - 8x$. This notation emphasizes the fact that the values of y, the **dependent variable**, depend on the values of x, the **independent variable**.

To evaluate a function, or find a functional value, means to substitute a given value in the domain into the equation to find the corresponding element in the range.

Example **Given $f(x) = x^2 + 2x$, find each value.**

a. $f(3)$

$f(x) = x^2 + 2x$ Original function

$f(3) = 3^2 + 2(3)$ Substitute.

$= 15$ Simplify.

b. $f(5a)$

$f(x) = x^2 + 2x$ Original function

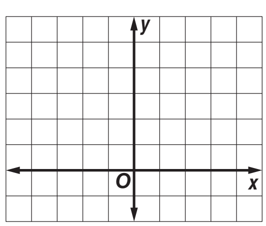
$f(5a) = (5a)^2 + 2(5a)$ Substitute.

$= 25a^2 + 10a$ Simplify.

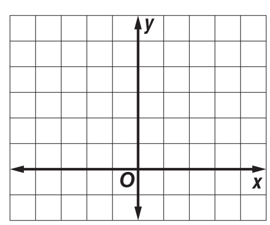
Exercises

Graph each relation or equation and determine the domain and range. Determine whether the relation is a *function*, is *one-to-one*, *onto*, *both*, or *neither*. Then state whether it is *discrete* or *continuous*.

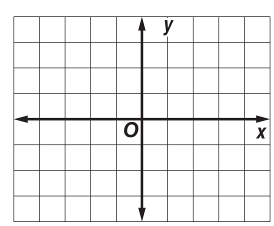
1. $y = 3$



2. $y = x^2 - 1$



3. $y = 3x + 2$



Find each value if $f(x) = -2x + 4$.

4. $f(12)$

5. $f(6)$

6. $f(2b)$

Find each value if $g(x) = x^3 - x$.

7. $g(5)$

8. $g(-2)$

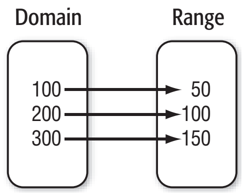
9. $g(7c)$

2-1 Skills Practice

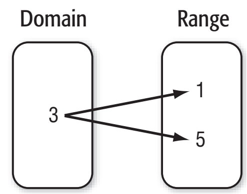
Relations and Functions

State the domain and range of each relation. Then determine whether each relation is a *function*. If it is a function, determine if it is *one-to-one*, *onto*, *both* or *neither*.

1.



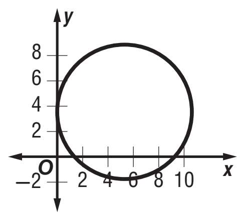
2.



3.

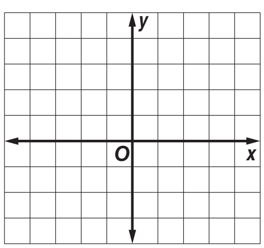
x	*y*
1	2
2	4
3	6

4.

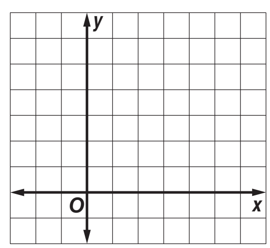


Graph each relation or equation and determine the domain and range. Determine whether the eqation is a *function*, is *one-to-one*, *onto*, *both*, or *neither*. Then state whether it is discrete or continuous.

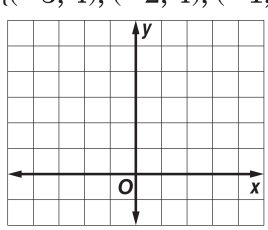
5. $\{(2, -3), (2, 4), (2, -1)\}$



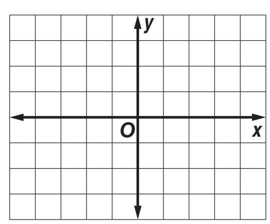
6. $\{(2, 6), (6, 2)\}$



7. $\{(-3, 4), (-2, 4), (-1, -1), (3, -1)\}$



8. $x = -2$



Find each value if $f(x) = 2x - 1$ and $g(x) = 2 - x^2$.

9. $f(0)$

10. $f(12)$

11. $g(4)$

12. $f(-2)$

13. $g(-1)$

14. $f(d)$

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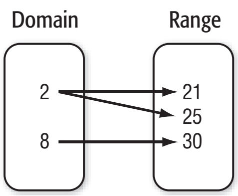
2-1 Practice

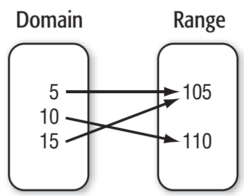
Relations and Functions

State the domain and range of each relation. Then determine whether each relation is a *function*. If it is a function, determine if it is *one-to-one*, *onto*, *both* or *neither*.

1.



2.



3.

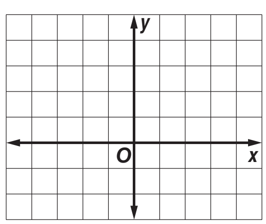
x	y
–3	0
–1	–1
0	0
2	–2
3	4

4.

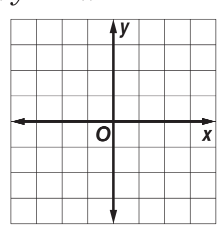
x	y
–2	–1
–2	1
–1	0
1	0
2	1

Graph each equation and determine the domain and range. Determine whether the relation is a *function*, is *one-to-one*, *onto*, *both*, or *neither*. Then state whether it is *discrete* or *continuous*.

5. $x = -1$



6. $y = 2x - 1$



Find each value if $f(x) = \dfrac{5}{x + 2}$ and $g(x) = -2x + 3$.

7. $f(3)$

8. $f(-4)$

9. $g\left(\frac{1}{2}\right)$

10. $f(-2)$

11. $g(-6)$

12. $f(m - 2)$

13. MUSIC The ordered pairs (1, 16), (2, 16), (3, 32), (4, 32), and (5, 48) represent the cost of buying various numbers of CDs through a music club. Identify the domain and range of the relation. Is the relation discrete or continuous? Is the relation a function?

14. COMPUTING If a computer can do one calculation in 0.0000000015 second, then the function $T(n) = 0.0000000015n$ gives the time required for the computer to do n calculations. How long would it take the computer to do 5 billion calculations?

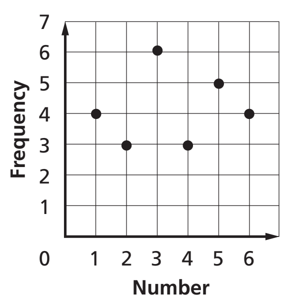
2-1 Word Problem Practice

Relations and Functions

1. PLANETS The table below gives the mean distance from the Sun and orbital period of the eight major planets in our Solar System. Think of the mean distance as the domain and the orbital period as the range of a relation. Is this relation a function? Explain.

Planet	Mean Distance from Sun (AU)	Orbital Period (years)
Mercury	0.387	0.241
Venus	0.723	0.615
Earth	1.0	1.0
Mars	1.524	1.881
Jupiter	5.204	11.75
Saturn	9.582	29.5
Uranus	19.201	84
Neptune	30.047	165

2. PROBABILITY Martha rolls a number cube several times and makes the frequency graph shown. Write a relation to represent this data.



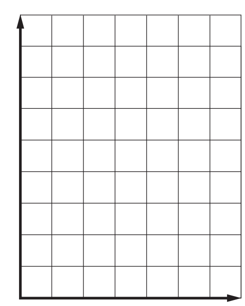
3. SCHOOL The number of students N in Vassia's school is given by $N = 120 + 30G$, where G is the grade level. Is 285 in the range of this function?

4. FLOWERS Anthony decides to decorate a ballroom with $r = 3n + 20$ roses, where n is the number of dancers. It occurs to Anthony that the dancers always come in pairs. That is, $n = 2p$, where p is the number of pairs. What is r as a function of p?

5. SALES Cool Athletics introduced the new Power Sneaker in one of their stores. The table shows the sales for the first 6 weeks.

Week	1	2	3	4	5	6
Pairs Sold	8	10	15	22	31	44

a. Graph the data.



b. Identify the domain and range.

c. Is the relation a function? Explain.

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2-1 Enrichment

Real Number Relations and Functions

It is interesting to think about one-to-one and onto correspondences between different subsets of the real numbers.

Example 1 **Is there one-to-one and onto correspondence between natural numbers and integers?**

Set the two groups up as follows:

1	2	3	4	5	6	7	8	9 . . .
↓	↓	↓	↓	↓	↓	↓	↓	↓
0	1	–1	2	–2	3	–3	4	–4 . . .

Even though the natural numbers are a subset of the integers, there is one unique natural number for each integer. The two sets do have a one-to-one correspondence.

Example 2 **Is there one-to-one and onto correspondence between integers and rational numbers?**

$\cdots -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \ldots$

$\cdots -\frac{1}{4} \quad -\frac{1}{3} \quad -\frac{1}{2} \quad -\frac{1}{1} \quad -\frac{2}{4} \quad -\frac{2}{3} \quad -\frac{2}{2} \quad -\frac{2}{1} \quad -\frac{3}{4} \quad -\frac{3}{3} \quad -\frac{3}{2} \quad -\frac{3}{1}$

No matter how you arrange the sets, there is never one unique integer for each rational number. There is not a one-to-one and onto correspondence.

Exercises

1. Is there a one-to-one and onto correspondence between the whole numbers and the natural numbers?

2. Is there a one-to-one and onto correspondence between the whole numbers and the integers?

3. Is there a one-to-one and onto correspondence between the natural numbers and the real numbers?

4. Is there a one-to-one and onto correspondence between the negative integers and the set of all integers?