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1-4 Study Guide and Intervention

Solving Absolute Value Equations

Absolute Value Expressions The **absolute value** of a number is its distance from 0 on a number line. The symbol $|x|$ is used to represent the absolute value of a number x.

Absolute Value	• **Words**	For any real number a, if a is positive or zero, the absolute value of a is a. If a is negative, the absolute value of a is the opposite of a.
	• **Symbols**	For any real number a, $\|a\| = a$, if $a \geq 0$, and $\|a\| = -a$, if $a < 0$.

Example 1 **Evaluate $|-4| - |-2x|$ if $x = 6$.**

$$\begin{aligned} |-4| - |-2x| &= |-4| - |-2 \cdot 6| \\ &= |-4| - |-12| \\ &= 4 - 12 \\ &= -8 \end{aligned}$$

Example 2 **Evaluate $|2x - 3y|$ if $x = -4$ and $y = 3$.**

$$\begin{aligned} |2x - 3y| &= |2(-4) - 3(3)| \\ &= |-8 - 9| \\ &= |-17| \\ &= 17 \end{aligned}$$

Exercises

Evaluate each expression if $w = -4$, $x = 2$, $y = \frac{1}{2}$, and $z = -6$.

1. $|2x - 8|$

2. $|6 + z| - |-7|$

3. $5 + |w + z|$

4. $|x + 5| - |2w|$

5. $|x| - |y| - |z|$

6. $|7 - x| + |3x|$

7. $|w - 4x|$

8. $|wz| - |xy|$

9. $|z| - 3|5yz|$

10. $5|w| + 2|z - 2y|$

11. $|z| - 4|2z + y|$

12. $10 - |xw|$

13. $|6y + z| + |yz|$

14. $3|wx| + \frac{1}{4}|4x + 8y|$

15. $7|yz| - 30$

16. $14 - 2|w - xy|$

17. $|2x - y| + 5y$

18. $|xyz| + |wxz|$

19. $z|z| + x|x|$

20. $12 - |10x - 10y|$

21. $\frac{1}{2}|5z + 8w|$

22. $|yz - 4w| - w$

23. $\frac{3}{4}|wz| + \frac{1}{2}|8y|$

24. $xz - |xz|$

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1-4 Study Guide and Intervention *(continued)*

Solving Absolute Value Equations

Absolute Value Equations Use the definition of absolute value to solve equations containing absolute value expressions.

For any real numbers a and b, where $b \geq 0$, if $|a| = b$ then $a = b$ or $a = -b$.

Always check your answers by substituting them into the original equation. Sometimes computed solutions are not actual solutions.

Example **Solve $|2x - 3| = 17$. Check your solutions.**

Case 1

$$\begin{aligned} a &= b \\ 2x - 3 &= 17 \\ 2x - 3 + 3 &= 17 + 3 \\ 2x &= 20 \\ x &= 10 \end{aligned}$$

CHECK

$$\begin{aligned} |2x - 3| &= 17 \\ |2(10) - 3| &\stackrel{?}{=} 17 \\ |20 - 3| &\stackrel{?}{=} 17 \\ |17| &\stackrel{?}{=} 17 \\ 17 &= 17 \checkmark \end{aligned}$$

Case 2

$$\begin{aligned} a &= -b \\ 2x - 3 &= -17 \\ 2x - 3 + 3 &= -17 + 3 \\ 2x &= -14 \\ x &= -7 \end{aligned}$$

CHECK

$$\begin{aligned} |2x - 3| &= 17 \\ |2(-7) - 3| &\stackrel{?}{=} 17 \\ |-14 - 3| &\stackrel{?}{=} 17 \\ |-17| &\stackrel{?}{=} 17 \\ 17 &= 17 \checkmark \end{aligned}$$

There are two solutions, 10 and -7.

Exercises

Solve each equation. Check your solutions.

1. $|x + 15| = 37$

2. $|t - 4| - 5 = 0$

3. $|x - 5| = 45$

4. $|m + 3| = 12 - 2m$

5. $|5b + 9| + 16 = 2$

6. $|15 - 2k| = 45$

7. $5n + 24 = |8 - 3n|$

8. $|8 + 5a| = 14 - a$

9. $\frac{1}{3}|4p - 11| = p + 4$

10. $|3x - 1| = 2x + 11$

11. $\left|\frac{1}{3}x + 3\right| = -1$

12. $40 - 4x = 2|3x - 10|$

13. $5f - |3f + 4| = 20$

14. $|4b + 3| = 15 - 2b$

15. $\frac{1}{2}|6 - 2x| = 3x + 1$

16. $|16 - 3x| = 4x - 12$

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1-4 Skills Practice

Solving Absolute Value Equations

Evaluate each expression if $w = 0.4$, $x = 2$, $y = -3$, and $z = -10$.

1. $|5w|$

2. $|-9y|$

3. $|9y - z|$

4. $-|17z|$

5. $-|10z - 31|$

6. $-|8x - 3y| + |2y + 5x|$

7. $25 - |5z + 1|$

8. $44 + |-2x - y|$

9. $2|4w|$

10. $3 - |1 - 6w|$

11. $|-3x - 2y| - 4$

12. $6.4 + |w - 1|$

Solve each equation. Check your solutions.

13. $|y + 3| = 2$

14. $|5a| = 10$

15. $|3k - 6| = 2$

16. $|2g + 6| = 0$

17. $10 = |1 - c|$

18. $|2x + x| = 9$

19. $|p - 7| = -14$

20. $2|3w| = 12$

21. $|7x - 3x| + 2 = 18$

22. $4|7 - y| - 1 = 11$

23. $|3n - 2| = \frac{1}{2}$

24. $|8d - 4d| + 5 = 13$

25. $-5|6a + 2| = -15$

26. $|k| + 10 = 9$

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1-4 Practice

Solving Absolute Value Equations

Evaluate each expression if $a = -1$, $b = -8$, $c = 5$, and $d = -1.4$.

1. $|6a|$

2. $|2b + 4|$

3. $-|10d + a|$

4. $|17c| + |3b - 5|$

5. $-6|10a - 12|$

6. $|2b - 1| - |-8b + 5|$

7. $|5a - 7| + |3c - 4|$

8. $|1 - 7c| - |a|$

9. $-3|0.5c + 2| - |-0.5b|$

10. $|4d| + |5 - 2a|$

11. $|a - b| + |b - a|$

12. $|2 - 2d| - 3|b|$

Solve each equation. Check your solutions.

13. $|n - 4| = 13$

14. $|x - 13| = 2$

15. $|2y - 3| = 29$

16. $7|x + 3| = 42$

17. $|3u - 6| = 42$

18. $|5x - 4| = -6$

19. $-3|4x - 9| = 24$

20. $-6|5 - 2y| = -9$

21. $|8 + p| = 2p - 3$

22. $|4w - 1| = 5w + 37$

23. $4|2y - 7| + 5 = 9$

24. $-2|7 - 3y| - 6 = -14$

25. $2|4 - n| = -3n$

26. $5 - 3|2 + 2w| = -7$

27. $5|2r + 3| - 5 = 0$

28. $3 - 5|2d - 3| = 4$

29. WEATHER A thermometer comes with a guarantee that the stated temperature differs from the actual temperature by no more than 1.5 degrees Fahrenheit. Write and solve an equation to find the minimum and maximum actual temperatures when the thermometer states that the temperature is 87.4 degrees Fahrenheit.

30. OPINION POLLS Public opinion polls reported in newspapers are usually given with a margin of error. For example, a poll with a margin of error of $\pm 5\%$ is considered accurate to within plus or minus 5% of the actual value. A poll with a stated margin of error of 63% predicts that candidate Tonwe will receive 51% of an upcoming vote. Write and solve an equation describing the minimum and maximum percent of the vote that candidate Tonwe is expected to receive.

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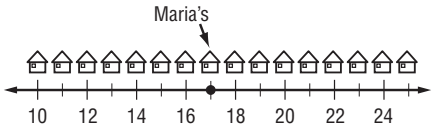
1-4 Word Problem Practice

Solving Absolute Value Equations

1. LOCATIONS Identical vacation cottages, equally spaced along a street, are numbered consecutively beginning with 10. Maria lives in cottage #17. Joshua lives 4 cottages away from Maria. If n represents Joshua's cottage number, then $|n - 17| = 4$. What are the possible numbers of Joshua's cottage?



2. HEIGHT Sarah and Jessica are sisters. Sarah's height is s inches and Jessica's height is j inches. Their father wants to know how many inches separate the two. Write an equation for this difference in such a way that the result will always be positive no matter which sister is taller.

3. AGES In 2005, 24.8% of all Americans were under 18 years old. Rhonda conducts a survey of the ages of students in eleventh grade at her school. On November 1, she finds the average age is 200 months. She also finds that two-thirds of the students are within 6 months of the average age. Write and solve an equation to determine the age limits for this group of students. How many months will it be until the first of these students turn 18?

4. TOLERANCE Martin makes exercise weights. For his 10-pound dumbbells, he guarantees that the actual weight of his dumbbells is within 0.1 pound of 10 pounds. Write and solve an equation that describes the minimum and maximum weight of his 10-pound dumbbells.

5. WALKING Jim is walking along a straight line. An observer watches him. If Jim walks forward, the observer records the distance as a positive number, but if he walks backward, the observer records the distance as a negative number. The observer has recorded that Jim has walked a, then b, then c feet.

a. Write a formula for the total distance that Jim walked.

b. The equation you wrote in part **a** should not be $T = |a + b + c|$. What does $|a + b + c|$ represent?

c. When would the formula you wrote in part **a** give the same value as the formula shown in part **b**?

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1-4 Enrichment

Considering All Cases in Absolute Value Equations

You have learned that absolute value equations with one set of absolute value symbols have two cases that must be considered. For example, $|x + 3| = 5$ must be broken into $x + 3 = 5$ or $-(x + 3) = 5$. For an equation with two sets of absolute value symbols, four cases must be considered.

Consider the problem $|x + 2| + 3 = |x + 6|$. First we must write the equations for the case where $x + 6 \geq 0$ and where $x + 6 < 0$. Here are the equations for these two cases:

$|x + 2| + 3 = x + 6$

$|x + 2| + 3 = -(x + 6)$

Each of these equations also has two cases. By writing the equations for both cases of each equation above, you end up with the following four equations:

$x + 2 + 3 = x + 6$ $\qquad$ $x + 2 + 3 = -(x + 6)$

$-(x + 2) + 3 = x + 6$ $\qquad$ $-x - 2 + 3 = -(x + 6)$

Solve each of these equations and check your solutions in the original equation, $|x + 2| + 3 = |x + 6|$. The only solution to this equation is $-\frac{5}{2}$.

Exercises

Solve each absolute value equation. Check your solution.

1. $|x - 4| = |x + 7|$

2. $|2x + 9| = |x - 3|$

3. $|-3x - 6| = |5x + 10|$

4. $|x + 4| - 6 = |x - 3|$

5. How many cases would there be for an absolute value equation containing three sets of absolute value symbols?

6. List each case and solve $|x + 2| + |2x - 4| = |x - 3|$. Check your solution.

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1-4 Spreadsheet Activity

Absolute Value Statements

You can use a spreadsheet to try several different values in an equation to help you determine whether the statement is *sometimes*, *always*, or *never* true. Remember that showing that a statement is true for some values does not prove that it is true for all values. However, finding one value for which a statement is false proves that it is not true for all values.

Determine whether $c|a + b| = |ca + cb|$ is *sometimes*, *always*, or *never* true.

Try a number of values for a, b, and c to determine whether the statement is true or false for each set of values.

Step 1 Use Columns A, B, and C for the values of a, b, and c. Choose several sets of values including positive and negative numbers, and zero.

Step 2 Use Column D to test the equation. A formula such as C2*ABS(A2+B2) = ABS(C2*A2+C2*B2) in cell D2 returns TRUE if the equation is true.

	A	B	C	D
1	a	b	c	c|a + b| = |ca + cb|
2	1	1	0	TRUE
3	-1	–1	–1	FALSE
4	2	2	2	TRUE
5	1	1	1	TRUE
6	1	1	–2	FALSE
7	2	1	0	TRUE
8	1	1	2	TRUE
9	0	0	3	TRUE
10				

Sheet 1 Sheet 2 Sheet 3

Through observation of Column D, when c is negative the statement is not true. The absolute value statement, $c|a + b| = |ca + cb|$ is sometimes true; it is true only if $c \geq 0$.

Exercises

Use a spreadsheet to determine whether each absolute value statement is *sometimes*, *always*, or *never* true.

1. For all real numbers a and b, $a \neq 0$, $|ax + b| = 0$.

2. If a and b are real numbers, then $|a + b| = |a| = |b|$.

3. If a and b are real numbers, then $|a + b| = -x$.

4. If a and b are real numbers, then $|a| - |b| = a - b$.

5. If a, b, and c are real numbers, then $c|a + b| = c|a| + |b|$.