

1-3 Study Guide and Intervention**Solving Equations**

Verbal Expressions and Algebraic Expressions The chart suggests some ways to help you translate word expressions into algebraic expressions. Any letter can be used to represent a number that is not known.

Word Expression	Operation
and, plus, sum, increased by, more than	addition
minus, difference, decreased by, less than	subtraction
times, product, of (as in $\frac{1}{2}$ of a number)	multiplication
divided by, quotient	division

Example 1 Write an algebraic expression to represent 18 less than the quotient of a number and 3.

$$\frac{n}{3} - 18$$

Example 2 Write a verbal sentence to represent $6(n - 2) = 14$.

Six times the difference of a number and two is equal to 14.

Exercises

Write an algebraic expression to represent each verbal expression.

- the sum of six times a number and 25
- four times the sum of a number and 3
- 7 less than fifteen times a number
- the difference of nine times a number and the quotient of 6 and the same number
- the sum of 100 and four times a number
- the product of 3 and the sum of 11 and a number
- four times the square of a number increased by five times the same number
- 23 more than the product of 7 and a number

Write a verbal sentence to represent each equation.

- $3n - 35 = 79$
- $2(n^3 + 3n^2) = 4n$
- $\frac{5n}{n + 3} = n - 8$

1-3 Study Guide and Intervention *(continued)*

Solving Equations

Properties of Equality To solve equations, we can use properties of equality.

Addition and Subtraction Properties of Equality	For any real numbers a , b , and c , if $a = b$, then $a + c = b + c$ and $a - c = b - c$.
Multiplication and Division Properties of Equality	For any real numbers a , b , and c , if $a = b$, then $a \cdot c = b \cdot c$ and, if $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$.

Example 1 Solve $10 - 8x = 50$.

$10 - 8x = 50$	Original equation
$10 - 8x - 10 = 50 - 10$	Subtract 10 from both sides.
$-8x = 40$	Simplify.
$x = -5$	Divide both sides by -8 .

Example 2 Solve $4x + 5y = 100$ for y .

$4x + 5y = 100$	Original equation
$4x + 5y - 4x = 100 - 4x$	Subtract $4x$ from both sides.
$5y = 100 - 4x$	Simplify.
$y = \frac{1}{5}(100 - 4x)$	Divide both sides by 5.
$y = 20 - \frac{4}{5}x$	Apply the distributive property.

Exercises

Solve each equation. Check your solution.

- | | | |
|---------------------------------|-----------------------------|-------------------------|
| 1. $3s = 45$ | 2. $17 = 9 - a$ | 3. $5t - 1 = 6t - 5$ |
| 4. $\frac{2}{3}m = \frac{1}{2}$ | 5. $7 - \frac{1}{2}x = 3$ | 6. $-8 = -2(z + 7)$ |
| 7. $0.2b = 10$ | 8. $3x + 17 = 5x - 13$ | 9. $5(4 - k) = -10k$ |
| 10. $120 - \frac{3}{4}y = 60$ | 11. $\frac{5}{2}n = 98 - n$ | 12. $4.5 + 2p = 8.7$ |
| 13. $4n + 20 = 53 - 2n$ | 14. $100 = 20 - 5r$ | 15. $2x + 75 = 102 - x$ |

Solve each equation or formula for the specified variable.

- | | |
|---------------------------------------|---|
| 16. $a = 3b - c$, for b | 17. $\frac{s}{2t} = 10$, for t |
| 18. $h = 12g - 1$, for g | 19. $\frac{3pq}{r} = 12$, for p |
| 20. $2xy = x + 7$, for x | 21. $\frac{d}{2} + \frac{f}{4} = 6$, for f |
| 22. $3(2j - k) = 108$, for j | 23. $3.5s - 42 = 14t$, for s |
| 24. $\frac{m}{n} + 5m = 20$, for m | 25. $4x - 3y = 10$, for y |

1-3 Skills Practice**Solving Equations****Write an algebraic expression to represent each verbal expression.**

1. 4 times a number, increased by 7 2. 8 less than 5 times a number
3. 6 times the sum of a number and 5 4. the product of 3 and a number, divided by 9
5. 3 times the difference of 4 and a number
6. the product of -11 and the square of a number

Write a verbal sentence to represent each equation.

7. $n - 8 = 16$ 8. $8 + 3x = 5$
9. $b + 3 = b^2$ 10. $\frac{y}{3} = 2 - 2y$

Name the property illustrated by each statement.

11. If $a = 0.5b$, and $0.5b = 10$, then $a = 10$. 12. If $d + 1 = f$, then $d = f - 1$.
13. If $-7x = 14$, then $14 = -7x$. 14. If $(8 + 7)r = 30$, then $15r = 30$.

Solve each equation. Check your solution.

15. $4m + 2 = 18$ 16. $x + 4 = 5x + 2$
17. $3t = 2t + 5$ 18. $-3b + 7 = -15 + 2b$
19. $-5x = 3x - 24$ 20. $4v + 20 - 6 = 34$
21. $a - \frac{2a}{5} = 3$ 22. $2.2n + 0.8n + 5 = 4n$

Solve each equation or formula for the specified variable.

23. $I = prt$, for p 24. $y = \frac{1}{4}x - 12$, for x
25. $A = \frac{x + y}{2}$, for y 26. $A = 2\pi r^2 + 2\pi rh$, for h

1-3 Practice**Solving Equations****Write an algebraic expression to represent each verbal expression.**

1. 2 more than the quotient of a number and 5 2. the sum of two consecutive integers
3. 5 times the sum of a number and 1 4. 1 less than twice the square of a number

Write a verbal sentence to represent each equation.

5. $5 - 2x = 4$ 6. $3y = 4y^3$
7. $3c = 2(c - 1)$ 8. $\frac{m}{5} = 3(2m + 1)$

Name the property illustrated by each statement.

9. If $t - 13 = 52$, then $52 = t - 13$. 10. If $8(2q + 1) = 4$, then $2(2q + 1) = 1$.
11. If $h + 12 = 22$, then $h = 10$. 12. If $4m = -15$, then $-12m = 45$.

Solve each equation. Check your solution.

13. $14 = 8 - 6r$ 14. $9 + 4n = -59$
15. $\frac{3}{4} - \frac{1}{2}n = \frac{5}{8}$ 16. $\frac{5}{6}c + \frac{3}{4} = \frac{11}{12}$
17. $-1.6r + 5 = -7.8$ 18. $6x - 5 = 7 - 9x$
19. $5(6 - 4v) = v + 21$ 20. $6y - 5 = -3(2y + 1)$

Solve each equation or formula for the specified variable.

21. $E = mc^2$, for m 22. $c = \frac{2d + 1}{3}$, for d
23. $h = vt - gt^2$, for v 24. $E = \frac{1}{2}Iw^2 + U$, for I

25. **GEOMETRY** The length of a rectangle is twice the width. Find the width if the perimeter is 60 centimeters. Define a variable, write an equation, and solve the problem.

26. **GOLF** Luis and three friends went golfing. Two of the friends rented clubs for \$6 each. The total cost of the rented clubs and the green fees for each person was \$76. What was the cost of the green fees for each person? Define a variable, write an equation, and solve the problem.

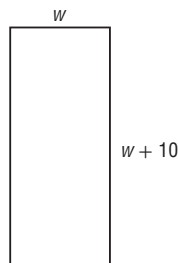
1-3 Word Problem Practice

Solving Equations

1. AGES Robert's father is 5 years older than 3 times Robert's age. Let Robert's age be denoted by R and let Robert's father's age be denoted by F . Write an equation that relates Robert's age and his father's age.

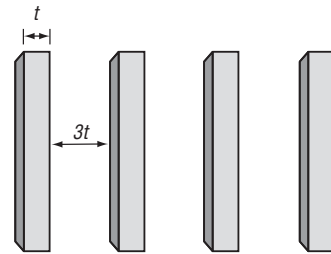
2. AIRPLANES The Citation Sovereign is a small jet that can carry up to 2650 pounds. The number of passengers p and the number of suitcases c that the airplane can carry are estimated by the equation $180p + 60c = 2650$. If 10 people board the aircraft, how many suitcases can the airplane carry?

3. GEOMETRY The length of a rectangle is 10 units longer than its width. If the total perimeter of the rectangle is 44 units, what is the width?



4. SAVINGS Jason started with d dollars in his piggy bank. One week later, Jason doubled the amount in his piggy bank. Another week later, Jason was able to add \$20 to his piggy bank. At this point, the piggy bank had \$50 in it. What is d ?

5. DOMINOES Nancy is setting up a train of dominos from the front entrance straight down the hall to the kitchen entrance. The thickness of each domino is t . Nancy places the dominos so that the space separating consecutive dominos is $3t$. The total distance that N dominos takes up is given by $d = t(4N + 1)$.



a. Nancy measures her dominos and finds that $t = 1$ centimeter. She measures the distance of her hallway and finds that $d = 321$ centimeters. Rewrite the equation that relates d , t , and N with the given values substituted for t and d .

b. How many dominos did Nancy have in her hallway?

1-3 Enrichment***United States' Gross National Product***

The Gross National Product, GNP, is an important indicator of the U.S. economy. The GNP contains information about the inflation rate, the Bond market, and the Stock market. It is composed of consumer goods, investments, government expenditures, exports, and imports.

Calculated from $GNP = C + I + G + X - M$, where

C is consumer goods (e.g. TVs, cars, food, furniture, clothes, doctors' fees, and dining)

I is investments (e.g. factories, computers, airlines, and housing)

G is government spending and investments (e.g. ships, roads, schools, NASA, and bombs)

X is exports (e.g. corn, wheat, cars, and computers)

M is for imports, (e.g. cars, computer chips, clothes, and oil)

$X - M$ is exports minus imports and equals trade surplus or deficit.

1. The most important sector of the U.S. economy is consumption. It makes up about 60% of the entire GNP. In 2000, the U.S.'s GNP was 10.5 trillion dollars. In the same year, there were 1 trillion dollars in investments, but a 1 trillion dollar trade deficit. Assuming that consumption made up 60% of the GNP, how much did the government budget for spending?
2. In 2001, the U.S. trade deficit remained at 1 trillion dollars, investments also remained steady at 1 trillion dollars. However consumption dipped to only 50% of the GNP, which increased to 12 trillion dollars. What was the effect on government spending? What might have caused the change?
3. If the GNP remains steady, and so do investments and government spending, but the trade deficit increases (to say 2 or 3 trillion dollars), what does this say about the consumption level?
4. Determine if there is a trade surplus or deficit when there is 12 trillion dollar GNP, 2 trillion in investments, 3 trillion in government investments, and 5 trillion in consumption. Explain why this situation may be favorable.

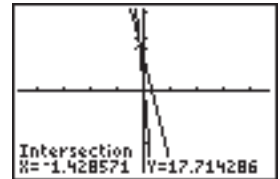
1-3 Graphing Calculator Activity

Solving Equations and Checking Solutions

When solving equations, checking the solutions is an important process. A graphing calculator can be used to check the solution of an equation.

Example 1 Solve $-2(5x - 1) - x = -4(x - 3)$.

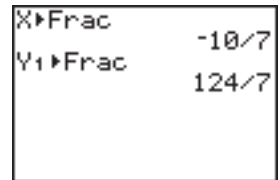
Graph the expression on the left side of the equation in **Y1** and the expression on the right side of the equation in **Y2**. Choose an appropriate view window so that the intersection of the graphs is visible. Then use the intersect command to find the coordinates of the common point.



[-47, 47] scl: 10 by [-31, 31] scl: 10

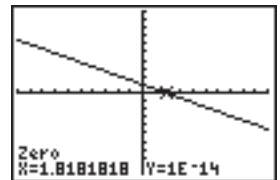
Keystrokes: $Y=$ $(-)$ 2 $($ 5 X,T,θ,n $-$ 1 $)$ $-$ X,T,θ,n $ENTER$ $(-)$ 4 $($ X,T,θ,n $-$ 3 $)$ $ZOOM$ 6 $ZOOM$ 8 $ENTER$ 2^{nd} $[CALC]$ 5 $ENTER$ $ENTER$ $ENTER$ 2^{nd} $[QUIT]$ X,T,θ,n $MATH$ $ENTER$ $ENTER$.

The x -coordinate, $-\frac{10}{7}$, is the solution to the equation. The y -coordinate is the value of both sides of the equation when $x = -\frac{10}{7}$.



Example 2 Solve $\frac{x}{5} - \frac{x}{4} = \frac{1}{2}(x - 2)$.

Graph the expression on the left side of the equation in **Y1** and the expression on the right side of the equation in **Y2**. Enter **Y1 - Y2** in **Y3**. Then graph the function in **Y3**. Use the zero function under the **CALC** menu to determine where the graph of **Y3** equals zero. This point will be the solution.



[-10, 10] scl: 1 by [-10, 10] scl: 1

Keystrokes: $Y=$ $($ X,T,θ,n \div 5 $)$ $-$ $($ X,T,θ,n \div 4 $)$ $ENTER$ $($ 1 \div 2 $)$ $($ X,T,θ,n $-$ 2 $)$ $ENTER$ $VARS$ \blacktriangleright $ENTER$ $ENTER$ $-$ $VARS$ \blacktriangleright $ENTER$ 2 \blacktriangle \blacktriangle \blacktriangleleft $ENTER$ \blacktriangle $ENTER$ $ZOOM$ 6 2^{nd} $[CALC]$ 2.

Use arrow keys and enter to set the bound prompts. The solution is $x = \frac{20}{11}$.

Exercises

Solve each equation.

- $-3(2w - 7) = 9 - 2(5w + 4)$
- $1.5(4 - x) = 1.3(2 - x)$
- $\frac{1}{4}(a + 2) = \frac{1}{6}(5 - a)$
- $3(2z + 25) - 2(z - 1) = 78$
- $\frac{m - 4}{3} - \frac{3m - 1}{5} = 1$
- $\frac{x + 5}{2} + \frac{1}{2} = 2x - \frac{x - 3}{8}$