NAME ______________________ DATE __________ PERIOD __________

1-2 Study Guide and Intervention

Properties of Real Numbers

Real Numbers All real numbers can be classified as either rational or irrational. The set of rational numbers includes several subsets: natural numbers, whole numbers, and integers.

R	real numbers	{all rationals and irrationals}
Q	rational numbers	{all numbers that can be represented in the form $\frac{m}{n}$, where m and n are integers and n is not equal to 0}
I	irrational numbers	{all nonterminating, nonrepeating decimals}
Z	integers	{…, −3, −2, −1, 0, 1, 2, 3, …}
W	whole numbers	{0, 1, 2, 3, 4, 5, 6, 7, 8, …}
N	natural numbers	{1, 2, 3, 4, 5, 6, 7, 8, 9, …}

Example **Name the sets of numbers to which each number belongs.**

a. $-\frac{11}{3}$ rationals (Q), reals (R)

b. $\sqrt{25}$

$\sqrt{25} = 5$ naturals (N), wholes (W), integers (Z), rationals (Q), reals (R)

Lesson 1-2

Exercises

Name the sets of numbers to which each number belongs.

1. $\frac{6}{7}$

2. $-\sqrt{81}$

3. 0

4. 192.0005

5. 73

6. $34\frac{1}{2}$

7. $\frac{\sqrt{36}}{9}$

8. 26.1

9. π

10. $\frac{15}{3}$

11. $-4.\overline{17}$

12. $\frac{\sqrt{25}}{2}$

13. −1

14. $\sqrt{42}$

15. −11.2

16. $-\frac{8}{13}$

17. $\frac{\sqrt{5}}{2}$

18. $33.\overline{3}$

19. 894,000

20. −0.02

1-2 Study Guide and Intervention *(continued)*

Properties of Real Numbers

Properties of Real Numbers

Real Number Properties		
	For any real numbers *a*, *b*, and *c*	
Property	**Addition**	**Multiplication**
Commutative	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity	$a + 0 = a = 0 + a$	$a \cdot 1 = a = 1 \cdot a$
Inverse	$a + (-a) = 0 = (-a) + a$	$a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a$, $a \neq 0$.
Closure	$a + b$ is a real number.	$a \cdot b$ is a real number.
Distributive	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	

Example **Simplify $9x + 3y + 12y - 0.9x$.**

$9x + 3y + 12y - 0.9x$	$= 9x + (-0.9x) + 3y + 12y$	Commutative Property (+)
	$= (9 + (-0.9))x + (3 + 12)y$	Distributive Property
	$= 8.1x + 15y$	Simplify.

Exercises

Simplify each expression.

1. $8(3a - b) + 4(2b - a)$

2. $40r + 18t - 5t + 11r$

3. $\frac{1}{5}(4j + 2k - 6j + 3k)$

4. $10(6g + 3h) + 4(5g - h)$

5. $12\left(\frac{a}{3} - \frac{b}{4}\right)$

6. $8(2.4r - 3.1t) - 6(1.5r + 2.4t)$

7. $4(20 - 4p) - \frac{3}{4}(4 - 16p)$

8. $5.5j + 8.9k - 4.7k - 10.9j$

9. $1.2(7x - 5y) - (10y - 4.3x)$

10. $9(7d - 4f) - 0.6(d + 5f)$

11. $2.5(12m - 8.5p)$

12. $\frac{3}{4}p - \frac{1}{5}r - \frac{3}{5}r - \frac{1}{2}p$

13. $4(10g + 80h) - 20(10h - 5g)$

14. $2(15d + 45c) + \frac{5}{6}(12d + 18c)$

15. $(7y - 2.1x)3 + 2(3.5x - 6y)$

16. $\frac{2}{3}(18m - 6p + 12m + 3p)$

17. $14(j - 2k) - 3j(4 - 7k)$

18. $50(3a - b) - 20(b - 2a)$

1-2 Skills Practice

Properties of Real Numbers

Name the sets of numbers to which each number belongs.

1. 34

2. -525

3. 0.875

4. $\frac{12}{3}$

5. $-\sqrt{9}$

6. $\sqrt{30}$

Name the property illustrated by each equation.

7. $3 \cdot x = x \cdot 3$

8. $3a + 0 = 3a$

9. $2(r + w) = 2r + 2w$

10. $2r + (3r + 4r) = (2r + 3r) + 4r$

11. $5y\left(\frac{1}{5y}\right) = 1$

12. $15x(1) = 15x$

13. $0.6[25(0.5)] = [0.6(25)]0.5$

14. $(10b + 12b) + 7b = (12b + 10b) + 7b$

Find the additive inverse and multiplicative inverse for each number.

15. 15

16. 1.25

17. $-\frac{4}{5}$

18. $3\frac{3}{4}$

Simplify each expression.

19. $3x + 5y + 2x - 3y$

20. $x - y - z + y - x + z$

21. $-(3g + 3h) + 5g - 10h$

22. $a^2 - a + 4a - 3a^2 + 1$

23. $3(m - z) + 5(2m - z)$

24. $2x - 3y - (5x - 3y - 2z)$

25. $6(2w + v) - 4(2v + 1w)$

26. $\frac{1}{3}(15d + 3c) - \frac{1}{2}(8c - 10d)$

NAME ______________________ DATE ____________ PERIOD ____________

1-2 Practice

Properties of Real Numbers

Name the sets of numbers to which each number belongs.

1. 6425 **2.** $\sqrt{7}$ **3.** 2π **4.** 0

5. $\sqrt{\frac{25}{36}}$ **6.** $-\sqrt{16}$ **7.** -35 **8.** -31.8

Name the property illustrated by each equation.

9. $5x \cdot (4y + 3x) = 5x \cdot (3x + 4y)$

10. $7x + (9x + 8) = (7x + 9x) + 8$

11. $5(3x + y) = 5(3x + 1y)$

12. $7n + 2n = (7 + 2)n$

13. $3(2x)y = (3 \cdot 2)(xy)$

14. $3x \cdot 2y = 3 \cdot 2 \cdot x \cdot y$

15. $(6 + -6)y = 0y$

16. $\frac{1}{4} \cdot 4y = 1y$

17. $5(x + y) = 5x + 5y$

18. $4n + 0 = 4n$

Find the additive inverse and multiplicative inverse for each number.

19. 0.4

20. -1.6

21. $-\frac{11}{16}$

22. $5\frac{5}{6}$

Simplify each expression.

23. $5x - 3y - 2x + 3y$

24. $-11a - 13b + 7a - 3b$

25. $8x - 7y - (3 - 6y)$

26. $4c - 2c - (4c + 2c)$

27. $3(r - 10t) - 4(7t + 2r)$

28. $\frac{1}{5}(10a - 15b) + \frac{1}{2}(8b + 4a)$

29. $2(4z - 2x + y) - 4(5z + x - y)$

30. $\frac{5}{6}\left(\frac{3}{5}x + 12y\right) - \frac{1}{4}(2x - 12y)$

31. TRAVEL Olivia drives her car at 60 miles per hour for t hours. Ian drives his car at 50 miles per hour for $(t + 2)$ hours. Write a simplified expression for the sum of the distances traveled by the two cars.

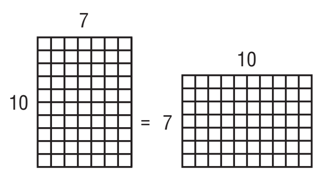
32. NUMBER THEORY Use the properties of real numbers to tell whether the following statement is true or false: If a and $b \neq 0$ and $a > b$, it follows that $a\left(\frac{1}{a}\right) > b\left(\frac{1}{b}\right)$. Explain your reasoning.

1-2 Word Problem Practice

Properties of Real Numbers

1. MENTAL MATH There are more than 3 million elementary teachers in the U.S. When teaching their students to multiply and learn place value, teachers often show that $54 \times 8 = (50 + 4) \times 8 = (50 \times 8) + (4 \times 8)$. What property is used?

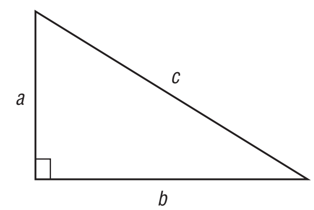
2. MODELS What property of real numbers is illustrated by the figure below?



3. VENN DIAGRAMS Make a Venn diagram that shows the relationship between natural numbers, integers, rational numbers, irrational numbers, and real numbers.

4. NUMBER THEORY Consider the following two statements.
I. The product of any two rational numbers is always another rational number.
II. The product of two irrational numbers is always irrational.
Determine if these statements are always, sometimes, or never true. Explain.

5. RIGHT TRIANGLES The lengths of the sides of the right triangle shown are related by the formula $c^2 = a^2 + b^2$.



For each set of values for a and b, determine the value of c. State whether c is a natural number.

a. $a = 5, b = 12$

b. $a = 7, b = 14$

c. $a = 7, b = 24$

Lesson 1-2

1-2 Enrichment

Properties of a Group

A set of numbers forms a group with respect to an operation if for that operation the set has (1) the Closure Property, (2) the Associative Property, (3) a member which is an identity, and (4) an inverse for each member of the set.

Example 1 **Does the set {0, 1, 2, 3, …} form a group with respect to addition?**

Closure Property: For all numbers in the set, is $a + b$ in the set? $0 + 1 = 1$, and 1 is in the set; $0 + 2 = 2$, and 2 is in the set; and so on. The set has closure for addition.

Associative Property: For all numbers in the set, does $a + (b + c) = (a + b) + c$? $0 + (1 + 2) = (0 + 1) + 2$; $1 + (2 + 3) = (1 + 2) + 3$; and so on. The set is associative for addition.

Identity: Is there some number, i, in the set such that $i + a = a = a + i$ for all a? $0 + 1 = 1 = 1 + 0$; $0 + 2 = 2 = 2 + 0$; and so on. The identity for addition is 0.

Inverse: Does each number, a, have an inverse, a′, such that $a' + a = a + a' = i$? The integer inverse of 3 is −3 since $-3 + 3 = 0$, and 0 is the identity for addition. But the set does not contain −3. Therefore, there is no inverse for 3.

The set is not a group with respect to addition because only three of the four properties hold.

Example 2 **Is the set {−1, 1} a group with respect to multiplication?**

Closure Property: $(-1)(-1) = 1$; $(-1)(1) = -1$; $(1)(-1) = -1$; $(1)(1) = 1$
The set has closure for multiplication.

Associative Property: $(-1)[(-1)(-1)] = (-1)(1) = -1$; and so on
The set is associative for multiplication.

Identity: $1(-1) = -1$; $1(1) = 1$
The identity for multiplication is 1.

Inverse: −1 is the inverse of −1 since $(-1)(-1) = 1$, and 1 is the identity. 1 is the inverse of 1 since $(1)(1) = 1$, and 1 is the identity. Each member has an inverse.

The set {−1, 1} is a group with respect to multiplication because all four properties hold.

Exercises

Tell whether the set forms a group with respect to the given operation.

1. {integers}, addition

2. {integers}, multiplication

3. $\left\{\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \cdots\right\}$, addition

4. {multiples of 5}, multiplication

5. $\{x, x^2, x^3, x^4, \cdots\}$ addition

6. $\{\sqrt{1}, \sqrt{2}, \sqrt{3}, \cdots\}$, multiplication

7. {irrational numbers}, addition

8. {rational numbers}, addition